The Impact of Discretionary Disclosure on Financial Reporting Systems: an Extension of Bayesian Persuasion*

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Abstract: We consider how a firm's design of its financial reporting system may be impacted by subsequent receipt and discretionary disclosure of private information. The firm seeks to induce posterior expectations that maximize the probability of meeting a crucial threshold. In the absence of private information, the firm prefers an imperfectly informative reporting system, notwithstanding that a perfectly informative reporting system is costless. Anticipating private information may cause the firm either to increase or decrease the reporting system's informativeness, depending on prior beliefs and the informativeness of private signals. Enforcing mandatory disclosure of private information may either increase or decrease the firm's welfare. However, regardless of the impact of private information on the design of the financial reporting system, the introduction of private information makes the firm unambiguously worse off.

Key Words: financial reporting, Bayesian persuasion, discretionary disclosure

I. INTRODUCTION

Firms make choices regarding the properties of their financial reporting systems through the accounting policies that they adopt and the information they gather. Accounting policy choices may reflect conservative or liberal biases, and information gathered for reporting purposes may be distorted toward data that align users' actions with firm preferences. Firms may also receive private information through the normal course of business. We are interested in how the properties of private information that the firm may receive, and the discretion over its disclosure that the firm may have, influence the firm's ex ante design of its financial reporting system. As we will show, the prospect of receiving private information may induce design choices that either enhance or diminish the informativeness of the firm's financial reports relative to a benchmark of choices the firm would make absent anticipation of private information and discretion over its disclosure. If not considered, the presence of such effects may be a confound to regulators and empiricists in assessing the consequences of policy changes affecting the information environment on financial reporting systems. Recognition of bias or distortions induced by the anticipation of private information and its disclosure could lead to policies to undo those effects on financial reporting. As well, policies that foster or deter the dissemination of ex post private information may indirectly impact financial reporting, sometimes in surprising ways.

We explore the effect of the potential receipt and disclosure of private information on the *ex ante* design of a public reporting system in a setting where a firm's primary concern is to induce an outsider's beliefs to be sufficiently high, i.e., above a threshold. There are a number of situations that illustrate accounting choices that fall within generally accepted accounting principles and aid firms in meeting important thresholds. A manufacturing firm offering product warranties and facing a threshold based on expected earnings might choose to recognize revenue at the time of sale rather than defer revenue until claims are submitted (a policy choice). It could then limit the information it gathers for estimating claims by conducting tests less likely to reveal product defects, resulting in lower warranty accruals. Similarly, a construction firm might choose the percentage of completion method of accounting for a project without gathering information about causes of project failures or delays that raise estimates of future costs. A merchandising firm may choose liberal credit policies (technically an operating rather than reporting choice) without fully investigating the risks of non-collection. In our model, these types of choices reflect the firm's ability to design how reports, generated by its reporting system, reflect the economic states underlying its operating activities. In these cases, further information from warranty claims, cost realizations, or customer defaults might arise after financial reports are released and, if damaging, might not be disclosed.[2] The focus of our model is on how the prospect of receiving this subsequent information, through channels independent of the reporting system, influences the design of said reporting system.

Models like ours, examining the design of information systems, where the sender can commit to a design that is observable to the intended receivers, are referred to as Bayesian persuasion models by Kamenica and Gentzkow (2011).[3] Financial reporting systems fit the Bayesian persuasion framework reasonably well in the sense that firms have flexibility in choosing accounting policies that may advance their interests, and these policies are generally disclosed. The firm in our setting designs its reporting system primarily to meet a crucial threshold. The design of the reporting system is complicated by the fact that, subsequent to issuing financial reports, firms are likely to receive private information, the disclosure of which is discretionary. Press releases, public announcements, management forecasts, and supplemental SEC filings are among the more familiar conduits through which private information may be disclosed. Since disclosure or non-disclosure of this information may also contribute to the formation of posterior beliefs affecting whether thresholds are met, anticipation of receiving private information factors into the design of financial reporting systems. Our study focuses on this interaction. We also compare discretionary with mandatory disclosure of private information in assessing the value to the firm of the option to not disclose. Stepping back, we consider the value of private information *per se*, i.e., whether the firm is better or worse off when it may become privately informed.[4]

[2] A useful distinction can be made between the information that a firm chooses to gather and information arrival over which the firm lacks control. Such a distinction is present in the examples provided. In each case, private information pertaining to future payoffs over which the firm has discretion over its disclosure arrives after financial reports have been released.

[3] These types of models are relatively new in the literature and have also been studied, for example, in Duggan and Martinelli (2011), Gentzkow and Kamenica (2014), Michaeli (2014), Taneva (2014), Alonso and Camara (2014), and Wang (2013). Without being cast as persuasion, Goex and Wagenhoffer (2009) and Arya, Glover and Sivaramakrishnan (1997) also consider *ex ante* commitment to information gathering.

[4] Hedlund (2015) considers a setting in which the sender has private information at the time of choosing the signal structure. There, the mere choice of reporting system conveys information about the sender's

In a pure Bayesian persuasion context, firms facing threshold concerns may seek to dampen the informativeness of financial reports.[5] The basic idea is that the firm may increase the relative frequency of reports that are just good enough to meet the threshold by allowing imperfection in reporting of states that would exceed the threshold. For instance, suppose that a perfectly informative, but relatively infrequent, good report implied a state that strictly exceeded the threshold. By allowing a good report to also sometimes be generated in a state that (if perfectly observed) would not meet the threshold, the firm may be able to increase the relative frequency of a good report and still meet the threshold, but with higher *ex ante* probability. In a vernacular familiar to accountants, threshold concerns create an incentive for introducing a liberal bias into financial reporting systems. Biases motivated in response to threshold concerns can therefore manifest in liberal accounting policy choices.[6]

The addition of a subsequent stage at which firms may or may not disclose private information not encompassed by its financial reports influences the optimal design of financial reporting systems in a surprising way. We identify conditions under which firms choose more informative financial reporting systems that *reduce* the probability of meeting the thresholds in comparison to the case where firms do not expect to receive private information.[7] In another more intuitive case, discretionary disclosure of private information induces firms to choose less informative financial reporting systems to offset the anticipated effects of

private information. In our setting, the reporting system is set before the arrival of private information, at which point the firm and outsider have symmetric information.

[5]As shown in Kamenica and Gentzkow (2011), the driving force behind the interior solution is that the firm's payoff as a function the outsider's beliefs has both strictly convex and concave regions. If the payoff was globally convex (concave), then a perfectly informative (uninformative) reporting system would be optimal.

[6]Note that we are interested in thresholds in terms of outsiders' beliefs, not in terms of specific earnings targets. There is considerable empirical support regarding the importance of meeting such thresholds in avoiding losses. Kausar, Taffler, and Tan (2006) find that institutional investors tend to divest after going concern qualifications. Menon and Williams (2010) find negative market reactions to going concern qualifications in audit reports likely driven by the dependencies of exchange listings, debt terms, and financing on obtaining unqualified reports. Graham and Harvey (2001) find that credit ratings are a major concern for CFOs in capital structure decisions, while Kisgen (2006) notes that an inability to maintain high ratings may exclude institutions from holding bonds, trigger higher interest rates, etc., thereby affecting capital structure decisions. Beneish and Press (1993) find that violations of debt covenants lead to increases in interest rates, and in a later study Beneish and Press (1995) detect negative market reactions associated with such violations. Li et al. (2011) find indirect evidence that failing to pass goodwill impairment tests was a principal concern of firms given the negative impact of impairments on analysts' and market expectations.

[7]As we elaborate below, a design that provides more informative financial reports of states that exceed the threshold may be necessary to overcome the reduction in posterior expectations from non-disclosure of a private signal. Increasing the informativeness of the reports comes at a cost: a reduced frequency of reports that cause beliefs to meet the threshold.

information contained in such disclosures. We further find that the option to not disclose may or may not be valuable to the firm relative to mandatory disclosure (presuming that such mandatory disclosure could be enforced). Stepping back to consider the impact of private information on the firm's welfare, we find that the firm is better off without the potential to receive private information. This result holds for general distributions and payoff functions.

Many studies in accounting employ models of financial reporting systems with state-dependent asymmetric informativeness, or bias, similar to our model. Gigler and Hemmer (2001) show how a conservative bias may reduce pre-emptive voluntary disclosure, thereby mitigating the value of communication between managers and shareholders. While they seek to address the question of how reporting quality affects discretionary disclosure, we seek to address how the prospect of discretionary disclosure influences properties of public reporting systems. Kwon, Newman, and Suh (2001) consider optimal compensation arrangements in a moral hazard context with limited liability for which bad reports are less informative and good reports more informative of underlying bad and good states, respectively. Gigler et al. (2009) show how bias in a reporting system may make it more or less likely that a favorable or unfavorable signal accurately reports the underlying state in a setting where investment continuation decisions are at stake. Beyer (2013) considers an aggregate reporting system for a multi-segment firm that only reports losses and not gains in asset values. Such a system is less informative about gains in values, but is more informative about losses by comparison with a system that reports both since losses are not offset by gains. Friedman, Hughes, and Saouma (2016) portray effects of reporting biases on product market competition.

As is typical in models of discretionary disclosure (e.g., Verrecchia, 1983; Dye, 1985; Jung and Kwon, 1988), in equilibrium, a low-end pool is formed and private information that would lower posterior expectations is suppressed. Only signals that would raise posterior expectations above the prior expectations are disclosed. Of course, rational receivers would lower their expectations upon not observing a disclosure to take into account that the sender may have realized a low signal. The prospect of disclosures that would raise posterior expectations enhances the firm's ability to meet a threshold when the realized financial report alone would be insufficient, while the prospect of non-disclosure diminishes the firm's ability when the realized financial report alone would be sufficient. Anticipation of these uncertain prospects serve as constraints on maximizing the probability of meeting a threshold through the design of the financial reporting system. Eliminating discretion resolves uncertainty regarding whether a private signal has been received, but expands the set of messages that the firm may send. In either case, the constraints implied by uncertainty detract from achieving the highest probability of meeting the threshold.[8]

An issue that we suppress in our basic model is the prospect of *ex post* manipulation of financial reports. In the absence of some added friction or noise, we can ignore such biasing since rational receivers of those reports will undo their effects, making them irrelevant.[9] As for biases that cannot be undone, we extend the basic model to allow for probabilistically manipulating reports before they are disseminated and show that our results continue to hold as long as there are limitations on a firm's ability to manipulate. The important feature of the financial reporting system structure in our model is that one cannot completely undo the effects of *ex ante* design choices *ex post*. In a closely related study, Stocken and Verrecchia (2004) allow for *ex post* manipulation of reports in a model with an *ex ante* choice of financial reporting system precision and subsequent receipt of a private signal. The sender's ability to manipulate the report *ex post* may induce a less precise *ex ante* reporting system choice. In contrast, our paper focuses on *ex ante* choices that affect precision and bias and the sender's ability to exercise discretion over disclosure of a private signal. In our paper, the potential for discretionary disclosure can have a negative or positive effect on the informativeness of the reporting system chosen *ex ante*.

Similar to our study, Kamenica and Gentzkow (2011) consider how an optimal information system will be set when the sender (the firm in our case) is uncertain about the beliefs of a receiver (the outside party in our case). In our model, given that the firm does not know if and what private information it will observe, there is also uncertainty about the beliefs of the outside party at the stage in which the reporting system is set. However, the firm has partial control, because it can choose to disclose or withhold this private information. In this

[8] Our model allows for the financial reporting system to be perfectly informative. Accordingly, any level of informativeness that can be achieved with the addition of a private signal can be achieved through the design of the financial reporting system alone.

[9] We also ignore any out-of-pocket costs to increasing the informativeness of the financial reporting system; a perfectly informative system is feasible at no such cost. From a modeling perspective, such costs are often introduced as a means to obtain interior rather than corner solutions. In our model, out-of-pocket costs that prevent corner solutions are unnecessary and would merely obscure the following insight: that a less than maximally informative reporting system may be desirable as a means of inducing beliefs that meet a threshold with greater probability.

context, we show that the firm cannot, by discretionary disclosure of subsequently acquired private information, improve the likelihood of meeting the threshold beyond that achievable from the public reporting system alone; i.e., given a choice, the firm strictly prefers not to obtain private information.

Our study also relates closely to two streams of empirical literature. First, several studies document associations between properties of financial reports (e.g., earnings quality or complexity) and discretionary disclosure as represented by management forecasts or guidance (e.g., Ball et al., 2012; Francis, Nanda, and Olsson, 2008; Guay, Samuels, and Taylor, 2015; Gong, Li, and Xie, 2009; Lennox and Park, 2006). Overall, the average sign of the association between financial reporting quality and the frequency and accuracy of management forecasts varies across these studies. Our study provides a theoretical foundation for observing mixed empirical evidence as well as conditions that can help disentangle non-monotonic relations.

The second stream of related empirical work provides substantial support for firms utilizing accounting practices as devices for boosting the likelihood that thresholds will be met. Bartov, Gul, and Tsui (2001) find an association between discretionary accruals and audit report qualifications.[10] Press and Weintrop (1990) find that firms use accounting flexibility to meet debt covenants. Healy and Palepu (1990) find the opposite; however, Begley (1990) suggests that this could be an identification issue. Sweeney (1994) finds that firms approaching covenant violation early-adopt mandatory income-increasing changes and that firm's discretionary changes are increasing in default costs. Dichev and Skinner (2002) find that a large number of firms meet or beat covenants suggesting manipulation of reports upon which covenants are based. Kim and Kross (1998) find evidence of manipulation of loan loss provisions coincident with a change in bank capital standards. Ramanna and Watts (2012) find firms tend to use discretion in applying tests of goodwill impairment. Chen, Lethmathe and Soderstrom (2015), study the firm's reporting behavior when their objective is to meet a return level required to be accepted into a UN carbon emission program. Bonachi, Mara and Shalev (2015) find evidence consistent with parent firms accounting for business combinations under common control at fair value when their leverage is high and they have net

[10]Signed accruals have been a common workhorse for detecting earnings manipulation. We suggest that biased accruals could be an artifact of accounting policies chosen *ex ante* as well as a consequence of *ex post* manipulations. The former would appear to be more likely in a context where thresholds apply over multiple reporting periods.

covenants.

Summarizing our contributions to the accounting literature, to the best of our knowledge, we are the first to model the impact of *ex post* discretionary disclosure of private information on the *ex ante* design of public reporting systems. Notwithstanding a high level of abstraction, our model captures an incentive for biased financial reporting distinct from other incentives characterized in the literature. Among the insights from our results are the following: a more informative financial reporting system induced by discretionary disclosure of private information may *weaken* the effect of the reporting system in raising the probability of meeting a crucial threshold;[11] by reducing uncertainty about whether private information has been received, mandatory disclosure (if implementable) may be preferred by the firm to discretionary disclosure; and the firm is better off without the potential to receive private information. These results are robust to generalizations of the firm's objective function to accommodate other uses of information, and limited *ex post* manipulation of reports.

II. THE MODEL

There is a stochastic state that indirectly affects the payoff of a risk-neutral firm through the decisions made by an outside party. The players have common prior beliefs. The firm can influence the beliefs of the outsider through public reports and disclosure of private information. The accounting policies that comprise the firm's *ex ante* choice of its financial reporting system are publicly observable. We assume that the firm receives a private signal with a probability strictly less than one as in Dye (1985) and Jung and Kwon (1988) after the financial reporting system has been implemented. This probability and the distribution generating private signals are also common knowledge. Disclosed signals are credible and it is not possible to credibly communicate not having received a signal.

For analytic tractability, we adopt a binary state and reporting structure similar to Gigler and Hemmer (2001), Kwon et al. (2001), Bagnoli and Watts (2005), Smith (2007), Chen and Jorgensen (2012), Guo (2012), and Friedman et al. (2016); albeit in a different context. While parsimonious, the structure is adequate for depicting persuasive behavior on the part of the firm in choosing its reporting system. A similar binary structure for

[11] As mentioned earlier, the more intuitive case of discretionary disclosure of private information inducing a less informative financial reporting system is also possible.

the firm's private signal, if received, is sufficient for depicting the impact of discretion over disclosure on the reporting system design choice. We allow the firm to choose the properties of the reporting system but take the properties of the private signals as exogenous. This captures the idea that the firm has flexibility in designing its financial reporting system, but often cannot control the arrival and content of private information, and allows us to focus on the influence of the properties of the private signal on the design of the reporting system. In contrast, Gigler and Hemmer (2001) explore a setting in which the reporting system is fixed and the private information system is endogenously chosen. In order to focus on the impact of discretionary disclosure on the design of the financial reporting system, we assume a parameterization that preserves pooling of a low signal realization with non-receipt of a signal as a rational strategy.

Formally, the firm's random state is represented by $\theta \in \{H, L\}$ where H and L represent high and low values, respectively. We normalize values by setting $H = 1$ and $L = 0$. The outsider's threshold against which he compares posterior expectations is represented by $k \in (0, 1)$. Common prior beliefs are defined by $\alpha = \Pr(\theta = H)$.[12] We assume $\alpha < k$ to avoid the trivial case where the threshold is met even in the absence of additional information provided through reports and messages. The financial reporting system generates a report with the structure:

$$\begin{aligned} \Pr(r = g|\theta = H) &= \beta_H \in [0, 1] \\ \Pr(r = g|\theta = L) &= \beta_L \in [0, 1] \end{aligned}$$

where $\beta_H \geq \beta_L$. We define by B the feasible set of values for (β_H, β_L), i.e., $B \equiv \{\beta_H, \beta_L : \beta_H, \beta_L \in [0, 1], \beta_H \geq \beta_L\}$. The firm chooses $\boldsymbol{\beta} \equiv (\beta_H, \beta_L) \in B$ prior to potentially receiving a private signal, s. With probability $q \in (0, 1)$, the firm receives a non-empty private signal $s \in \{h, l\}$ with the following structure:

$$\begin{aligned} \Pr(s = h|\theta = H) &= \gamma_H \in [0, 1] \\ \Pr(s = h|\theta = L) &= \gamma_L \in [0, 1] \end{aligned}$$

where $\gamma_H > \gamma_L$, implying the private signal is informative. The firm cannot credibly communicate not having received a signal, $s = \emptyset$, which happens with probability $1 - q$. Upon

[12]In our model, beliefs are equivalent to the probability that the state is high, which, given our assumption of H = 1, is also the expected value. As such, we tend to use expectation and beliefs interchangeably.

receiving a non-empty signal, s, the firm can either truthfully disclose that signal by sending a message $m = s$ or not disclose, in which case it sends the same message, $m = \emptyset$, as when a signal is not received.

We assume that the firm's payoff is increasing in the outside party's posterior expectation about the firm's state. Of principal interest, the firm receives an additional benefit if the expected state meets or exceeds a threshold $k \in (0, 1)$. Formally, we define the firm's *ex post* payoff as:

$$\pi \equiv \omega_T \mathbb{1}_{E[\theta|r,m] \geq k} + \omega_C E[\theta|r, m],$$

where $\omega_T = 1$ is the discrete benefit (or loss avoided) from meeting the threshold, and $\omega_C > 0$ is the sensitivity of the *ex post* payoff to an increase in the posterior expectation. We discuss separately the case of $\omega_C = 0$.

The above expression introduces necessary elements for a theoretically interesting interior solution in a minimally-parameterized function that reflects the firm's desire for outsiders to have higher posterior beliefs about the underlying state. There are two components: a step-function component, $\omega_T \mathbb{1}_{E[\theta|m,r] \geq k}$; and a linear component, $\omega_C E[\theta|m, r]$. The step function gives the firm an incentive to set a reporting system that is informative but does not fully reveal the underlying state (i.e., an interior solution to the problem of setting the reporting system). The linear component provides important incentives in the discretionary disclosure stage of the game. As in Kamenica and Gentzkow (2011), obtaining an interior solution requires that the firm's *ex post* payoff should have both strictly convex and concave regions. This could be achieved by more general functions or by more densely-parameterized functions (e.g., piecewise linear with $n > 3$ pieces). However, our parameterization provides the necessary components while maintaining parsimony and tractability. For example, our two-component model concisely captures a firm whose stock price increases linearly in the market's average belief about its earnings, captured by θ, and experiences a jump when the expected growth is sufficiently high for the firm to be classified as a growth company and included in growth-based index funds. In Section IV, we extend the firm's objective function to incorporate an operational benefit from generating more informative reports.

Formally, we take the outsider as a passive Bayesian, but note that the firm's payoff function can be derived from an outsider who chooses two actions, $a_T, a_C \in [0, 1]$ to maximize a payoff function given by $u \equiv a_T(\theta - k) - (a_C - \theta)^2$. The outsider's optimal actions in this formulation are $\{\hat{a}_T, \hat{a}_C\} \in \arg\max_{a_T, a_C} E[u|r,m] = \{\mathbb{1}_{E[\theta|m,r] \geq k}, E[\theta|m,r]\}$. The threshold k here captures the relative importance of type-1 versus type-2 errors to the outsider. For example, high k implies that a false positive (i.e., choosing $a_T = 1$ when $\theta = L = 0$) is costlier than a false negative (i.e., choosing $a_T = 0$ when $\theta = H = 1$), and vice versa for low k. Under this specification the firm's payoff is given by $\pi \equiv \omega_T \hat{a}_T + \omega_C \hat{a}_C$, where ω_T and ω_C represent the sensitivities of the firm's payoff to the outside party's actions. In the context of the growth fund example, a_T represents inclusion in growth funds, a_C captures firm valuation, and k captures the benchmark that funds use to determine whether the firm is a "growth" firm.

From the Law of Iterated Expectations and $\omega_T = 1$, the firm's *ex ante* expected payoff is

$$\begin{aligned} E[\pi] &= E[\omega_T \mathbb{1}_{E[\theta|r,m] \geq k} + \omega_C E[\theta|r,m]] \\ &= \Pr(E[\theta|r,m] \geq k) + \omega_C \alpha. \end{aligned}$$

It is straightforward to show that maximizing the firm's expected payoff is equivalent to maximizing the probability of meeting the threshold, i.e.,

$$\arg\max_{\boldsymbol{\beta} \in B} E[\pi] = \arg\max_{\boldsymbol{\beta} \in B} \Pr(E[\theta|r,m] \geq k)$$

since $\omega_C \alpha$ is independent of the firm's reporting system choices.[13] When choosing across regimes and reporting systems we therefore focus only on the probability of meeting the threshold.

Figure 1 depicts the timeline of events. At date 1, the firm chooses the parameters $\boldsymbol{\beta} \equiv \{\beta_H, \beta_L\}$ governing the financial reporting system. The state, θ, is drawn by nature, but observed by neither the firm nor the outsider. At date 2, the financial report is realized and observed by both players, and either a private signal s is realized and privately observed only by the firm, or no signal is received. At date 3, the firm sends either a message $m = s$, or $m = \emptyset$ to the outside party. At date 4, the outside party forms a posterior expectation of the state and assesses whether the threshold has been met. The firm receives $\omega_C E[\theta|r,m]$ and an additional benefit ω_T normalized to 1 if the outsider's posterior expectations meet

[13]Even though the first term of the firm's payoff, $\omega_C \alpha$, does not matter for the design of the reporting system from an *ex ante* perspective, $\omega_C > 0$ *is* important for obtaining a unique disclosure strategy in the voluntary disclosure stage.

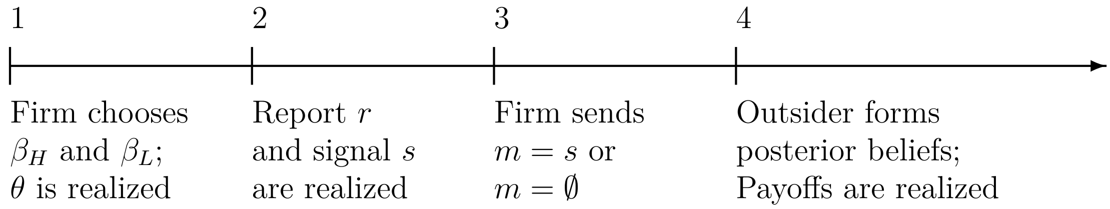


Figure 1: Timeline of events

or beat the threshold. In our analysis, $\boldsymbol{\beta}$ is chosen before the firm potentially observes the private signal. Otherwise, the choice of $\boldsymbol{\beta}$ would signal the firm's private information, as in Hedlund (2015). The timing of the public report, r, relative to the private signal, s, and the message, m, is inconsequential, as is the timing of the firm's choice of $\boldsymbol{\beta}$ relative to nature's unobserved draw of θ.

III. ANALYSIS

Financial Reporting Without Subsequent Receipt of Private Information

We first consider a special case in which the firm never receives private information. This case is a pure persuasion game in which posterior expectations are based only on the firm's financial report. Given that at date 1 the expected payoff to the firm is $E[\pi] = \Pr(E[\theta|r] \geq k) + \alpha\omega_C$, it is apparent that in setting the reporting system the firm seeks to maximize the probability that the threshold is met. Because $r \in \{g, b\}$ and $E[\theta] = \alpha < k$ by assumption, the firm can only meet the threshold if the reporting system generates a sufficiently informative good report, $r = g$. Consequently, the firm maximizes the probability of sending a good report, conditional on the good report inducing posterior beliefs that meet or exceed the threshold ("P" denotes pure persuasion):

$$\mathcal{P}(P): \qquad \max_{\beta \in B} \Pr(r = g) \quad \text{s.t.} \quad E[\theta|r = g] \geq k.$$

The solution to this program is $\beta_H^P = 1$ and $\beta_L^P = \frac{\alpha(1-k)}{k(1-\alpha)} \in (0, 1)$, implying a probability that the threshold is met of $\frac{\alpha}{k} > \alpha$. With perfect information the firm would meet the threshold less frequently, with a probability matching the prior of α. Both parties are rational and update consistent with Bayes' Rule, notwithstanding that the information provided by the firm's reporting system is slanted in a manner that serves the firm's interests.[14]

To understand the intuition for this result consider the extreme choices of $\boldsymbol{\beta}$. Setting $\beta_H = \beta_L$ implies an uninformative reporting system with no updating of beliefs. Hence, the outside party stays with prior belief $\alpha < k$, and the threshold is never met. At the other extreme, $\beta_H = 1$ and $\beta_L = 0$ imply a perfectly informative system. In this case, the outside party's posterior beliefs equal 1 if $r = g$, implying the threshold is exceeded, and 0 if $r = b$, implying the threshold is not met. It follows that the threshold is met with probability α.

Note that assurance of a high state given a good report is a stronger condition than is necessary to meet the threshold. The firm can increase the probability of meeting the threshold by allowing some good reports to be generated in low states. While this diminishes the posterior expectation given a good report, the expectation may still be sufficient to meet the threshold. This is accomplished by setting $\beta_H = 1$ and solving for β_L in the following expression:

$$E[\theta|r = g] \quad = \quad k,$$

where $E[\theta|r = g] = \frac{\alpha\beta_H}{\alpha\beta_H + (1-\alpha)\beta_L}$. This ensures that (i) upon observing a good report the beliefs of the outsider will just meet the required threshold k, and (ii) the odds of meeting the threshold are maximized.

Financial Reporting With Discretionary Disclosure of Private Information

We now consider the full fledged setting in which the firms may receive additional private information after the public reporting system has been set.

Optimization With Discretionary Disclosure

The possible receipt and discretionary disclosure of a private signal adds a second stage at which the firm makes a decision and the outside party updates beliefs. Accordingly, we

[14]The distribution over posterior beliefs (i.e., the outsider's expectation that the underlying state is high) generated by reports is as follows. The outsider has a posterior belief equal to k with probability $\frac{\alpha}{k}$ and a posterior belief of 0 with probability $\frac{k-\alpha}{k}$. We note that these posterior beliefs satisfy the law of iterated expectations; i.e., $\frac{k-\alpha}{k} \times 0 + \frac{\alpha}{k} \times k = \alpha$. This is equivalent to the "Bayesian plausibility" requirement in Kamenica and Gentzkow (2011). In our case, we incorporate this requirement in our calculations of posterior beliefs using Bayes' Rule rather than explicitly including the requirement as an additional constraint in the optimization programs.

solve the model by backward induction. Recall that the firm receives a benefit, $\omega_T = 1$, if and only if $E[\theta|r, m] \geq k$. Having normalized the states at $H = 1$ and $L = 0$, the above expectation is simply the posterior probability of $\theta = H$ given a report r and message m, i.e., $E[\theta|r, m] = Pr(\theta = H|r, m)$.

Suppose the firm receives a private signal s. Since $\gamma_H > \gamma_L$, the posterior probability of a high state is greater conditional on message $m = h$ than on $m = \emptyset$ and also greater conditional on message $m = \emptyset$ than on $m = l$. Given that in the main analysis we assume the firm always strictly benefits from higher posterior beliefs ($\omega_C > 0$), the lemma below follows immediately:[15]

Lemma 1 *The firm always discloses when $s = h$ and never discloses when $s = l$.*

Moving back to the choice of parameters governing the financial reporting system $\boldsymbol{\beta}$, as is the case without private information, the firm wants to maximize the probability that posterior beliefs meet or exceed the threshold. By Lemma 1, the firm always withholds low private signals. Hence, the reporting system and discretionary disclosure jointly create a set of four possible outcomes: $\{(r = g, m = h), (r = g, m = \emptyset), (r = b, m = h), (r = b, m = \emptyset)\}$. Clearly, the posterior expectations of the state in each of these four possible outcomes, $E[\theta|r, m] = \Pr(\theta = H|r, m)$, differ whenever $\beta_H > \beta_L$. Given that $\alpha < k$, $E[\theta|r = b, m = \emptyset] < \alpha < k$ always holds, but by adjusting the properties of the reporting system, the firm can induce posterior expectation to at least meet the threshold for other combinations of reports and messages. Specifically, there are four feasible combinations of the remaining three report-message pairs, since meeting the threshold with $(r = g, m = \emptyset)$ or $(r = b, m = h)$ implies meeting or beating the threshold with the more positive report-message pair, $(r = g, m = h)$. Each combination gives rise to a constrained optimization program for which some $\boldsymbol{\beta}$ is optimal. We solve the firm's problem by first solving for the optimal $\boldsymbol{\beta}$ that maximizes the probability of meeting or exceeding the threshold with a specific set of report-message combinations, and then determining which set, at its optimum, is best for the firm. The combinations and related programs are as follows, where "D" denotes discretionary disclosure:

[15]Except where indicated otherwise, proofs of all formal results are presented in the appendix.

$$
\begin{aligned}
\mathcal{P}1(D): \quad & \max_{\beta \in B} \Pr(r=g, m=h) + \Pr(r=b, m=h) + \Pr(r=g, m=\emptyset) \\
& \text{s.t.} \quad E[\theta | r=g, m=h] \geq k, \\
& \qquad\;\; E[\theta | r=b, m=h] \geq k, \\
& \qquad\;\; E[\theta | r=g, m=\emptyset] \geq k; \\
\mathcal{P}2(D): \quad & \max_{\beta \in B} \Pr(r=g, m=h) + \Pr(r=b, m=h) \\
& \text{s.t.} \quad E[\theta | r=g, m=h] \geq k, \\
& \qquad\;\; E[\theta | r=b, m=h] \geq k; \\
\mathcal{P}3(D): \quad & \max_{\beta \in B} \Pr(r=g, m=h) + \Pr(r=g, m=\emptyset) \\
& \text{s.t.} \quad E[\theta | r=g, m=h] \geq k, \\
& \qquad\;\; E[\theta | r=g, m=\emptyset] \geq k; \\
\mathcal{P}4(D): \quad & \max_{\beta \in B} \Pr(r=g, m=h) \\
& \text{s.t.} \quad E[\theta | r=g, m=h] \geq k.
\end{aligned}
$$

Elaborating on $\mathcal{P}1(D)$, the objective function is composed of the unconditional joint probability of report-message combinations including a good report and disclosure of a high signal, a bad report and disclosure of a high signal, and a good report and non-disclosure of a signal. The constraints ensure that the threshold is met for each combination. The first constraint (good report-high signal) will be slack, while at least one of the next two constraints (good report-no message or bad report-high signal) will bind, as each implies a reporting system in which the good report-high signal combination yields a posterior belief strictly higher than the threshold. Each of the next two programs, $\mathcal{P}2(D)$ and $\mathcal{P}3(D)$, considers two combinations of reports and messages while eliminating one of the potentially binding constraints in $\mathcal{P}1(D)$. $\mathcal{P}4(D)$ considers only one combination while eliminating both of the potentially binding constraints in $\mathcal{P}1(D)$, which allows the remaining constraint to bind. Eliminating constraints enlarges the feasible regions, but reduces the set of report-message combinations that result in posterior beliefs at or above the threshold. Hence, *a priori* we cannot say which program solution will provide the highest probability and related expected benefit of meeting the threshold for a given set of exogenous parameters. Solutions to the programs are provided in the Appendix.

Characteristics of Optimal Financial Reporting Systems

We begin this section by identifying a set of conditions on model parameters that determine which of the solutions to the above programs dominates. These conditions lead to characterizations of optimal financial reporting systems. We further assess the impact of discretionary disclosure and, separately, the potential availability of private information by comparing the optimal financial reporting system with the solution to the pure persuasion game benchmark.

Condition 1 $\gamma_H \geq \overline{g} \equiv \frac{1-\gamma_L kq}{q(2-\gamma_L q-k)}$

We refer to the above condition as capturing private signal informativeness. Note that the lower bound, $\overline{g}$, on the probability of a high signal given a high state in Condition 1 is increasing in the probability of a high signal given a low state, γ_L. Either an increase in γ_H or a decrease in γ_L widens the spread between those probabilities, which naturally captures private signal informativeness. Accordingly, we classify private signals as more informative if Condition 1 is satisfied and as less informative otherwise.

Condition 2 $\alpha > \frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$

Prior beliefs are said to be optimistic if Condition 2 is satisfied and pessimistic otherwise.

Proposition 1 *If Condition 1 is not satisfied (<u>less</u> informative private signals), then the firm's optimal financial reporting system is defined by* $1 = \beta_H^* > \beta_L^* > 0$ *and the threshold* k *is met or exceeded with a report* $r = g$ *independent of the message.*

A less informative private signal implies weak influence of messages on the outside party's posterior beliefs and, therefore, a primary reliance on the reporting system to induce favorable posterior beliefs. In this case, the solution of $\mathcal{P}3(D)$ is globally optimal, and the threshold is met following a good financial report irrespective of the message sent by the firm. In comparison with the solution to the benchmark pure persuasion game (equivalent to a completely uninformative private signal or $q = 0$), it is optimal for the firm to choose a financial reporting system that generates a somewhat more informative good report. This is accomplished by reducing the probability of a good report in a low state, $\beta_L^P > \beta_L^*$, while holding constant the probability of a good report in a high state $\beta_H^* = \beta_H^P = 1$. Although decreasing β_L implies a more informative good report, it also reduces the frequency of good reports, thereby lowering the unconditional probability of a report that induces a posterior expectation that meets the threshold. The former effect is necessary to allow the firm to meet the threshold with the combination of a good report and non-disclosure of a private signal. Although meeting the threshold with only a good report and a high signal as in $\mathcal{P}4(D)$ would allow the firm to increase the frequency of a good report, the joint unconditional probability of just this combination is lower, implying that the threshold would not be met as often.

As the next proposition establishes, increasing private signal informativeness to the point where Condition 1 is satisfied changes the way that the firm's private information affects its financial reporting system:

Proposition 2 *When Condition 1 is satisfied (<u>more</u> informative private signals):*

(i) If Condition 2 is not satisfied (pessimistic priors), then the firm's optimal financial reporting system is defined by $1 = \beta_H^{**} > \beta_L^{**} > 0$ *and both report* $r = g$ *and message* $m = h$ *are necessary to meet the threshold.*

(ii) If Condition 2 is satisfied (optimistic priors), then $1 > \beta_H^{***} > \beta_L^{***} > 0$ *and the threshold is met with* either *report* $r = g$ or *message* $m = h$.

Recall that Condition 1 is satisfied when private signals are more informative and Condition 2 is satisfied when prior beliefs are optimistic. It is useful to compare the solution in part (i) of Proposition 2 to the solution in Proposition 1 in assessing the effect of satisfying Condition 1. A more informative private signal under program $\mathcal{P}3(D)$ makes it more difficult to meet the threshold with the combination of a good report and non-disclosure of a signal. In other words, this combination implies a lower posterior belief, tightening the constraint on meeting the threshold for that combination due to a more informative low signal. As a consequence, the firm must choose a more informative but less frequent good report, which is accomplished by reducing the probability of a good report in a low state. However, the firm can do better in program $\mathcal{P}4(D)$, where a good report and a more informative high signal imply a higher posterior belief. This combination allows the firm to relax the constraint on meeting the threshold by choosing a less informative but more frequent good report, achievable by increasing the probability of a good report in a low state in comparison to the benchmark pure persuasion game, i.e., $\beta_L^P < \beta_L^{**}$.

When Conditions 1 and 2 are satisfied, having both optimistic prior beliefs and a more informative private signal makes it possible for the firm to meet the threshold with a combination of a bad report and high private signal. This is achieved by reducing the probability of a good report in a high state such that a bad report no longer implies a low state with certainty. Accordingly, under part (ii), the firm does best in program $\mathcal{P}1(D)$ where the threshold is met by a combination of a good report and any message or a bad report and a high message. In this case, the firm also reduces the probability of a good report in a low state in comparison to the benchmark pure persuasion game; i.e., $\beta_H^{***} < \beta_H^P = 1$ and $\beta_L^{***} < \beta_L^P$. While the firm can benefit from having a high message, in order to benefit when a low message is sent, it is crucial to make the bad report less than fully informative, which implies setting $\beta_H < 1$.

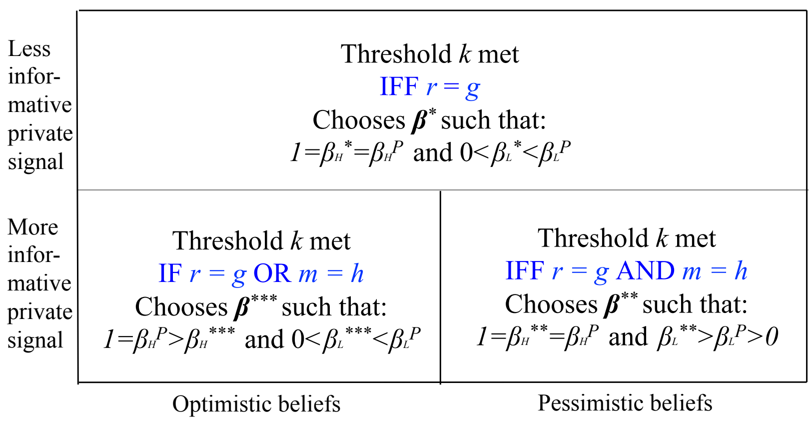


Figure 2: Firm's choice of financial reporting system defined by β_H and β_L

Firm's choice of financial reporting system defined by β_H and β_L. Stars indicate optima as described in Propositions 1 (*), 2 (i) (**) and 2 (ii) (***); $r \in \{g, b\}$ is the public report; $m \in \{s, \emptyset\}$ is the discretionary message based on the private signal $s \in \{h, l\}$; and β_H (β_L) is the probability of $r = g$ conditional on the state being high (low).

Figure 2 summarizes the results of Propositions 1 and 2. As we would anticipate, public financial reports and messages of private information are partial substitutes. Under pessimistic prior beliefs, less (more) informative private signals imply the choice of a more (less) informative financial reporting system i.e., $\beta_L^{**} > \beta_L^*$. The implication of more informative private signals for the informativeness of the financial reporting system in the remaining case of optimistic prior beliefs requires a measure of informativeness that encompasses both parameters β_H and β_L. For this case, we resort to the variance of expectations conditional on reports (i.e., $Var[E[\theta|r]]$) to show that less (more) informative private signals again imply the choice of a more (less) informative financial reporting system.[16] Stepping back to consider the informativeness of the combination of financial reports r and messages m using the variance of expectations conditional on all available information (i.e., $Var[E[\theta|r, m]]$) we find that more (less) precise private signals imply less (more) total information available to the outsiders.[17]

Value of Private Information

Comparing the expected payoff of the firm corresponding to the solutions in Proposition 1 and Proposition 2 with that in the pure persuasion game we see that the addition of a private signal to financial reports lowers the expected payoff. The firm still expects to do better, though, than it would if it provided a perfectly informative financial reporting system:

Corollary 1 *The expected payoff of the firm in the discretionary regime is: (i) always lower than the expected payoff under the pure persuasion benchmark; but (ii) higher than with either no reporting system or a perfectly informative reporting system.*

Corollary 1 follows directly from a comparison of the expected payoffs at the optimal $\boldsymbol{\beta}$ in each of the cases described in Proposition 1 and Proposition 2. An important feature underlying Corollary 1 is that the firm has unrestricted control over the properties of the reporting system. Through these properties, the firm essentially chooses the distribution of posterior beliefs over the states of nature induced by the report, constrained only by the fact that the Bayesian outsider's expected beliefs must equal his prior beliefs. In the benchmark case, the firm has complete control over this distribution. In contrast, when the firm has access to private information, it loses a degree of control because, for any report that it sends, multiple beliefs might be induced over which the firm must take expectations. Intuitively, it is possible for the firm in the benchmark case to choose signal properties that would induce the same distribution of posterior beliefs as would be induced by the

[16]See the Appendix for the proof. The variance of posterior expectations and equivalent measures have been used in prior studies as measures of information content (e.g., Michaeli, 2014; Friedman et al., 2016).

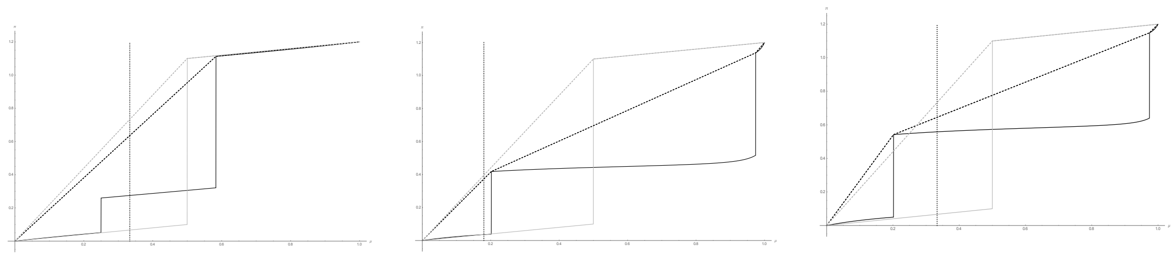
[17]This result follows from straightforward, but somewhat tedious algebra and is available from the authors.

combination of the reporting system and the potentially-disclosed message. However, since $\boldsymbol{\beta^P}$ is unique, adding private information and discretionary disclosure that does not replicate the distribution in the benchmark case reduces the firm's expected payoff. In Section IV we show that this intuition holds for general distributions and payoff functions.

To provide additional intuition for the driving forces behind this result let $\mu \equiv E[\theta|r]$ and $\Pi(\mu) \equiv E_m[\pi(\mu, m)]$, where the expectation is taken over the realizations of m. As shown in Kamenica and Gentzkow (2011) the expected payoff from an optimal reporting system depends on the concave closure of $\Pi(\mu)$ when the firm might have access to private information, and depends on the concave closure of $\pi(\mu)$ when the firm surely lacks such access. Kamenica and Gentzkow (2011) provide this result in terms of a receiver with beliefs that the sender does not know when designing the reporting system. In our setting, the firm's potential receipt and disclosure of private information causes it to be uncertain of the outsider's message-dependent beliefs at the time when the firm chooses $\boldsymbol{\beta}$. As defined above, $\pi(\mu)$ has a jump of $\omega_T = 1$ at the point where the posterior expectation based on the firm's report, $\mu(r)$, equals the threshold, k. $\Pi(\mu)$ has a jump of $\omega_T \cdot Pr(m = h)$ at the point where the posterior expectation based on the firm's report, $\mu(r)$, combined with a high message, $m = h$, equals k, and a further jump of $\omega_T \cdot Pr(m = \emptyset)$ at the point where the posterior, $\mu(r)$, combined with a null message, $m = \emptyset$, equals k. The total vertical distance of the two jumps in $\Pi(\mu)$ is equal to the vertical jump in $\pi(\mu)$, since the first step is $\omega_T \cdot Pr(m = h)$, the second step is $\omega_T \cdot Pr(m = \emptyset)$, and $\Pr(m = \emptyset) + \Pr(m = h) = 1$.

Similar to Kamenica and Gentzkow (2011), the maximum expected payoff, which is achieved with the optimal reporting system, is the concave closure of $\Pi(\mu)$ or $\pi(\mu)$ (depending on whether the firm might have private information) evaluated at the prior belief α. That is, our primary concern is the value of the concave closure of the payoff function, evaluated at α.

We first consider the case when Condition 1 is not satisfied (i.e., when the reporting system will be set according to Proposition 1). As illustrated by the numerical example in Figure 3(a), the concave closure of $\pi(\mu)$ is above the concave closure of $\Pi(\mu)$ evaluated at the prior belief, α. This implies that the firm's expected payoff is always lower when it has potential access to private information, compared to the case when it is known to be uninformed. Similarly, the numerical example in Figure 3(b) illustrates the case when



(a) Condition 1 is not satisfied

(b) Condition 1 is satisfied but Condition 2 is not satisfied

(c) Both Condition 1 and Condition 2 are satisfied

Figure 3: Comparison of the firm's expected payoffs in the pure persuasion benchmark (gray) and when the firm may have private information (black). Solid lines represent firm payoffs. Dashed lines are concave closures, and the vertical dotted line marks $\mu = \alpha$. For all panels $\omega_T = 1$; $\omega_C = 0.2$; $k = 0.5$; $\gamma_L = 0.25$. Further, $\{\alpha, \gamma_H, q\}$ are given as $\{0.33, 0.75, 0.5\}$ in panel (a); $\{0.18, 0.99, 0.99\}$ in panel (b); and $\{0.33, 0.99, 0.99\}$ in panel (c).

Condition 1 is satisfied but Condition 2 is not satisfied (i.e., when the reporting system will be set according to Proposition 2(i)). The numerical example in Figure 3(c) illustrates the case when both Condition 1 and Condition 2 are satisfied (i.e., when the reporting system will be set according to Proposition 2(ii)).

It is evident that discretionary disclosure of private information does not enhance the firm's ability to meet the threshold over what the firm could achieve with the financial reporting system alone, absent the potential receipt of private information. This implies that if the firm had control over the private information, it would choose never to receive such information (i.e., set $q = 0$) or choose a completely uninformative private information system so that outsiders ignore the message (i.e., set $\gamma_H = \gamma_L$). We formally state this result in the following Corollary:

Corollary 2

(i) If the firm could endogenously choose whether to have access to private information or not, it would choose never to have access to information ($q = 0$).

(ii) If the firm could endogenously choose the properties of the private signal, it would choose to receive an uninformative private signal ($\gamma_H = \gamma_L$).

However, an ability to forestall the receipt of private information would seem to be impossible given all of the ways in which information may arrive. It would appear to be similarly impossible to design commitments not to disclose information when there are potential benefits from influencing outsiders' beliefs ($\omega_C > 0$). Hence, if, as we would normally expect, the firm derives such a benefit, then the firm is stuck in an undesirable equilibrium.

Alignment of preferences for private information

It is apparent from the outsider's objective function that the outsider prefers greater precision in the combined information conveyed through the financial reporting system and private messages. Earlier we identified conditions on prior beliefs and private signal informativeness under which discretionary disclosure of private information induced either a more informative or less informative financial reporting system compared to the benchmark of a pure persuasion game. Extending the assessment of precision to the combined reports and messages, we observe that given low prior beliefs and more informative private signals the variance of posterior expectations is smaller compared to the variance in the pure persuasion game (implying lower precision of combined reports and messages). In this case, the preferences of the firm and the outsider are such that both would be better off without the potential receipt and disclosure of private information. In the remaining cases, preferences for the presence of private information are either misaligned or the alignment is ambiguous.[18]

Non-disclosure equilibrium

The firm's objective function in our model includes a benefit from higher posterior expectations implying a further role for the information contained in the financial report and the private signal. This captures the outsider's ability to choose actions beyond those that depend solely on assessing whether a threshold has been met. In this subsection, we consider the prospect of an alternative equilibrium when the firm's only objective is to avoid a loss from failing to meet the threshold. This is a special case of the model with $\omega_C = 0$.

Proposition 3 *With $\omega_C = 0$, there exists an equilibrium in which the firm never discloses its private signal.*

The formal proof is omitted as it follows from the discussion below. Suppose that the outside party believed that the firm would never disclose its private signal.[19] In order to sustain that

[18]The comparison of posterior expectations is available upon request.

[19]Any choice of betas other than those optimal for the pure persuasion game would indicate that the firm intended to make use of private information subsequently received. In this sense, the choice of betas serve as a "signal" to the outsider that private information will not be disclosed.

belief when the firm receives a high signal, the firm chooses the same (observable) financial reporting system as in the pure persuasion game. Following a good report, in the pure persuasion game, the outside party's posterior expectation is exactly high enough to just meet the threshold, and disclosing a high signal would induce a posterior expectation in excess of the threshold, which provides no further benefit to the firm. For a bad report, disclosing a high signal is moot since a bad report implies a bad state with certainty. Given that *ex ante* the solution to the pure persuasion game at least weakly dominates the solution under discretionary disclosure, the firm has no incentive to defect at either stage.

While we find this result interesting, the assumption of no further benefit to disclosure of a high signal is not descriptive of situations that firms actually face given the many other roles that have been ascribed to discretionary disclosure. Any marginal benefit to disclosure of a high private signal beyond that of meeting a crucial threshold, in and of itself, no matter how negligible, suffices to eliminate this equilibrium, notwithstanding that the firm may be better off with no disclosure.

Liberal Bias in Financial Reporting

We relate our results expressed in terms of $\boldsymbol{\beta}$ to a bias proxy, denoted, χ, as in Friedman et al. (2016) through the following transformation of variables:

$$\chi \equiv \frac{1 - \beta_L - \beta_H}{2}.$$

A positive value of χ connotes a conservative bias while a negative value connotes a liberal bias. The implied biases corresponding to the solutions in the benchmark pure persuasion case and Propositions 1 and 2 are liberal consistent with the tendency in all cases to increase the frequency of good reports while reducing their informativeness in order to produce the highest joint unconditional probability of meeting the threshold. Only the solution to Proposition 2(i) includes a liberal bias greater than that in the pure persuasion game; i.e., $\chi^{**} < \chi^P$. This is because, with pessimistic prior beliefs, the firm relies on both a good financial report and a more informative high private signal to meet the threshold. The latter allows the firm to further increase the unconditional probability of a good report by more liberally biasing the reporting system than in the pure persuasion game. In the other two cases, discretionary disclosure of private signals leads to less liberal biasing of the financial reporting system; i.e., $\chi^* > \chi^P$ and $\chi^{***} > \chi^P$. Supposing that regulators such as the

SEC and FASB may seek on general principles to induce more informative financial reporting, then this is advanced by less liberal (equivalently, more conservative) reporting in the sense of reducing the probability of a good report in a low state; i.e., decreasing β_L.

Ex post Manipulation of Financial Reports

In this subsection, we consider a setting in which the firm can successfully manipulate the report generated by the financial reporting system with positive probability.[20] We assume that the firm would *ex post* prefer to disseminate good reports; hence, manipulation takes the form of a bad report being portrayed as a good report with probability p. Suppose that an interim report $r^I = \{g', b'\}$ is generated with conditional probabilities

$$\Pr(r^I = g' | \theta = H) = \nu_H$$
$$\Pr(r^I = g' | \theta = L) = \nu_L.$$

If the interim report is $r^I = g'$, then the firm always sends a final report $r = g$. If the interim report is $r^I = b'$, then the firm sends a final report $r = g$ with probability p and sends a final report $r = b$ with probability $1 - p$. It follows that

$$\begin{aligned} \beta_H &= \Pr(r = g | \theta = H) = \nu_H + (1 - \nu_H)p \\ \beta_L &= \Pr(r = g | \theta = L) = \nu_L + (1 - \nu_L)p, \end{aligned}$$

where p is the exogenous parameter that captures the firm's probabilistic ability to bias interim reports upwards before disseminating them. In equilibrium, the firm will choose ν_H and ν_L such that the total state-dependent report probabilities match those implied by the optimal $\boldsymbol{\beta}$ derived above. As an example, consider the case of Proposition 1 when Condition 1 is not satisfied, implying $\beta_H^* = 1$ and $\beta_L^* = \frac{\alpha(1-k)(1-\gamma_H q)}{k(1-\alpha)(1-\gamma_L q)}$. The optimal ν_H^* and ν_L^* will satisfy

$$\nu_H^* + (1 - \nu_H^*)p = \beta_H^*$$
$$\nu_L^* + (1 - \nu_L^*)p = \beta_L^*.$$

[20] If a bad report could always be portrayed as a good report, reports would be uninformative of the underlying state, rendering Bayesian persuasion moot in the context of our model. The same would be true for general distributions. Noise such as our assumption of a probabilistic effect of manipulation both limits the effect of manipulation in expectation and prevents the receiver from being able to undo its effect.

Solving the system of equations, we obtain $\nu_H^* = 1$ and $\nu_L^* = \frac{\alpha(1-k)(1-\gamma_H q)}{k(1-\alpha)(1-\gamma_L q)(1-p)} - \frac{p}{1-p}$. From the expressions for the optimal ν's, note that ν_L^* is non-negative if and only if p is sufficiently small. If p is too high, then the firm can too-easily manipulate bad reports into good reports *ex post*, thereby weakening the informativeness of good reports to the point where the firm no longer meets the receiver's decision threshold when a good report is sent. In other words, if the firm can easily manipulate reports *ex post*, then it may not be able to set up an information system *ex ante* that will optimize its expected utility. If it is not too easy to manipulate reports *ex post*, then the ability to probabilistically manipulate bad reports into good reports influences the interim reporting system properties, but does not influence the state-conditional probabilities of reports provided to outsiders in total, leaving our qualitative results intact.

IV. EXTENSIONS

Discretionary vs. Mandatory Disclosure

We next consider whether the option to disclose or not disclose is beneficial to the firm. To do so, we solve for the optimal financial reporting system design through a series of programs similar to those in the previous section, but with the firm forced to disclose both low and high signals. We refer to this as a *mandatory* disclosure setting. While we abstract away from costs of mandatory disclosure, our results in this setting are suggestive of when a firm would seek to submit *ex ante* to certification of whether it *ex post* possessed private information.

Optimization With Mandatory Disclosure

The set of programs under mandatory disclosure are as follows, with solutions and programs denoted by "M":

$$\begin{aligned}
\mathcal{P}1(M): \quad & \max_{\beta \in B} \Pr(r = g, m = h) \\
& \text{s.t.} \quad E[\theta | r = g, m = h] \geq k; \\
\mathcal{P}2(M): \quad & \max_{\beta \in B} \Pr(r = g, m = h) + \Pr(r = g, m = \emptyset) \\
& \text{s.t.} \quad E[\theta | r = g, m = h] \geq k, \\
& \qquad\;\; E[\theta | r = g, m = \emptyset] \geq k;
\end{aligned}$$

$$
\begin{aligned}
\mathcal{P}3(M): \qquad & \max_{\beta \in B} \Pr(r=g, m=h) + \Pr(r=b, m=h) \\
& \text{s.t.} \quad E[\theta | r=g, m=h] \geq k, \\
& \qquad\quad E[\theta | r=b, m=h] \geq k; \\
\mathcal{P}4(M): \qquad & \max_{\beta \in B} \Pr(r=g, m=h) + \Pr(r=b, m=h) + \Pr(r=g, m=\emptyset) \\
& \text{s.t.} \quad E[\theta | r=g, m=h] \geq k, \\
& \qquad\quad E[\theta | r=b, m=h] \geq k, \\
& \qquad\quad E[\theta | r=g, m=\emptyset] \geq k; \\
\mathcal{P}5(M): \qquad & \max_{\beta \in B} \Pr(r=g, m=h) + \Pr(r=g, m=\emptyset) + \Pr(r=g, m=l) \\
& \text{s.t.} \quad E[\theta | r=g, m=h] \geq k, \\
& \qquad\quad E[\theta | r=g, m=\emptyset] \geq k, \\
& \qquad\quad E[\theta | r=g, m=l] \geq k; \\
\mathcal{P}6(M): \qquad & \max_{\beta} \Pr(r=g, m=h) + \Pr(r=g, m=\emptyset) \\
& \qquad + \Pr(r=g, m=l) + \Pr(r=b, m=h) \\
& \text{s.t.} \quad E[\theta | r=g, m=h] \geq k, \\
& \qquad\quad E[\theta | r=g, m=\emptyset] \geq k, \\
& \qquad\quad E[\theta | r=g, m=l] \geq k, \\
& \qquad\quad E[\theta | r=b, m=h] \geq k.
\end{aligned}
$$

Orderings of Disclosure Regimes

We next compare the discretionary and the mandatory disclosure regimes from the firm's point of view.

Proposition 4 *Suppose Condition 1 is not satisfied (less informative private signals), with γ_H sufficiently low, i.e., $\gamma_H < \min\{\overline{g}, g_o, g_{oo}\}$. Then, the firm strictly prefers discretion over mandatory disclosure.*

The applicable discretionary disclosure case for this parameterization is depicted in Proposition 1. Under mandatory disclosure, a low private signal is no longer pooled with non-receipt of a signal. As a consequence, a good report need not be as informative as under discretionary disclosure in order for the posterior beliefs following a combination of a good report and non-disclosure to meet the threshold. However, a good report that is only sufficiently informative to meet the threshold for that combination, when combined with a low signal, will not meet the threshold. If under mandatory disclosure the firm sought to meet the threshold for both non-receipt of a signal and a low signal, then the effect of having to compensate for a low signal in choosing a reporting system implies a worse solution than for $\mathcal{P}3(D)$. In the proof, we show that the expected value from $\mathcal{P}3(D)$ exceeds that from all programs $\mathcal{P}1\,(M) - \mathcal{P}6(M)$. Hence, discretion in this case is valuable to the firm.

Proposition 5 *Suppose Condition 1 is satisfied (more informative private signals), with* $\gamma_H > max\{\bar{g}, g^o, g^{oo}\}$. *Then,*

(i) if Condition 2 is not *satisfied (pessimistic priors), then the firm is indifferent between discretion and mandatory disclosure.*

(ii) if Condition 2 is satisfied (optimistic priors), then the firm strictly prefers mandatory disclosure to discretion.

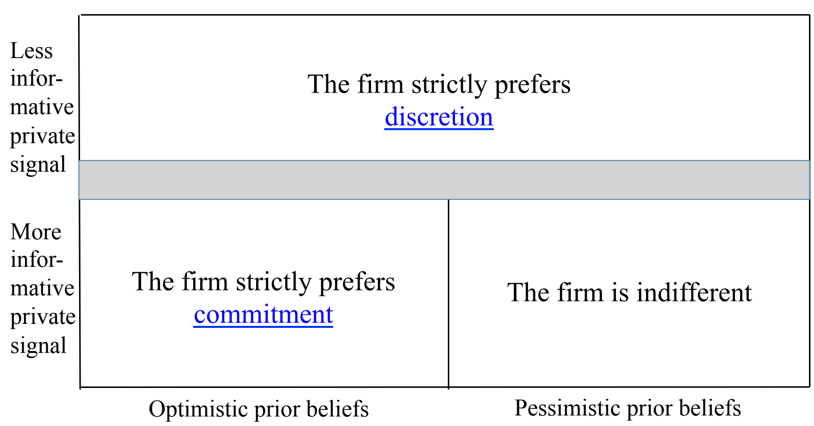


Figure 4: Firm's preference over regimes

The applicable discretionary disclosure cases for these parameterizations are $\mathcal{P}4(D)$ and $\mathcal{P}1(D)$, respectively. In part (i), we show that $\mathcal{P}1(M)$ is globally optimal. Since it corresponds to $\mathcal{P}4(D)$ by considering only the combination of a good report and high signal in meeting the threshold, the solutions are identical implying indifference by the firm between discretionary and mandatory disclosure. As for part (ii), we show that $\mathcal{P}4(M)$ brings a higher expected payoff to the firm than the optimal program with discretion, $\mathcal{P}1(D)$. While they both consider combinations of a good report and high signal, bad report and high signal, and good report and non-disclosure, the former does not pool non-receipt of a signal with a low signal, thereby making it possible to meet the threshold with a less informative but more frequent good report. Hence, the firm strictly prefers mandatory disclosure to discretion. Figure 4 depicts the regions in which the firm prefers either of the two regimes.

Although interesting as a benchmark in appreciating the consequences of discretion, mandatory disclosure may not be a realistic option given that the costs of monitoring compliance and enforcing penalties for non-compliance when a firm's receipt of information is uncertain are likely to be prohibitively high. Our results do suggest, though, that firms with optimistic priors who expect to receive informative private signals might be willing to pay for certification services that provide a mechanism for implementing an *ex ante* commitment to disclosure of potentially private information.

Further Use of Information

In this subsection, we consider additional benefits that the firm may have from information (public and private) by assuming that the firm's *ex post* payoff is

$$\pi^O = \lambda[-(\theta - d)^2] + (1-\lambda)[\omega_T \mathbb{1}_{E[\theta|r,m]\geq k} + \omega_C E[\theta|r,m]],$$

where d represents an operating decision (unobservable to the outsider) that the firm takes at date 3 after observing the financial report and the private signal and $\lambda \in [0,1]$ is the relative importance of the firm's operating decision.[21] The firm benefits from having the action, d, match the state, θ, implying a preference for more informative reports.

As $\lambda \to 0$, the firm only cares about the outsider's beliefs, i.e., we are back to the payoff considered in the main analysis. Alternatively, as $\lambda \to 1$, the firm only cares about its operating decision. In this case, the firm's *ex post* payoff collapses to $-(\theta - d)^2$ and the firm will therefore choose a perfectly informative reporting system: $\beta_H - \beta_L = 1$.

For any $\lambda \in (0,1)$ the firm would balance the operational benefit from implementing a

[21]Given that d is unobservable by the outsider, he updates beliefs only based on the report and the message.

more informative reporting system (and taking better operating decisions) with the objective of meeting the outsider's threshold. For sufficiently small λ, the firm would implement a more informative reporting system relative to the baseline model, but would not choose a perfectly informative one ($\beta_H - \beta_L < 1$). As λ increases, the operating decision becomes more important driving the firm to increase the precision of the reporting system. For λ sufficiently high, the operating decision will be sufficiently important for the firm to set a perfectly informative reporting system.

Value of Private Information for General Distributions and Payoffs

Recall that under certain conditions mandatory disclosure dominates discretionary disclosure from the firm's point of view. However, the solution to the pure persuasion game continues to be at least weakly preferred. In Section III, we showed that the firm is better off without private information under discretionary disclosure and binary distributions. In this subsection, we extend this result in three directions. First, we consider general distributions to establish that our earlier result is robust to the relaxation of the binary structure. Second, we consider a more general class of payoff functions. Third, we allow for mandatory as well as discretionary disclosure.

The intuition for the general result is similar to the intuition for Corollaries 1 and 2. Any distribution of posterior beliefs implementable with a combination of the financial report and disclosure of private information can be implemented through the design of the financial reporting system alone. The uncertainty associated with whether a private signal will be received and the realization of such a signal tends to induce distortions in the financial reporting system that diminish the firm's expected payoff.

Proposition 6 *The firm weakly prefers not to have access to private information, regardless of whether it has discretion over its disclosure or not. This preference is strict unless: (i) the private signal is uninformative; (ii) the firm can commit ex ante to never disclose the observed signal; (iii) the firm can fully commit ex ante to messages that are conditional on reports; or (iv) the firm prefers a perfectly informative reporting system in the absence of private information.*

The above results address a broader class of report-design problems by senders in choosing properties of public reporting systems than that considered in our main analysis. While we are principally concerned with the impact of discretionary disclosure of private signals following a public report, it follows from the proposition that any uncertainty introduced by signals or messages subsequent to such a report may detract from the sender's ability to maximize its expected benefits through the design of its public reporting system. The signals may be private to the sender as in our model, obtained by the receiver, or generated by nature. In all cases, the *ex ante* uncertainty as to their realization may interfere with the sender's ability to attain maximal utility in situations where the outsiders are Bayesian. Given a desire for an imperfectly-informative reporting system, informative private signals, and an inability to fully commit to report dependent messages, the sender is strictly better off with no private information from any source.

V. CONCLUSION

We consider the effects of discretionary disclosure of private information on financial reporting system design choices. Our model is an extension of Bayesian persuasion games in which a sender designs a reporting system *ex ante*, with the objective of maximizing the expectation of meeting an outsider's posterior beliefs threshold upon which the sender's welfare depends. The sender in the context of our model is a firm and the receiver is an outside party such as an auditor, credit rating agency, lender, investor, fund manager, or certifying agency, whose beliefs influence the firm's payoffs through, for example, audit opinions, debt ratings, debt covenant waivers or renegotiations, price-setting, or any of myriad certification requirements. While a perfectly informative reporting system is assumed to be feasible at no cost, the firm can do better with a less informative system that enhances the firm's odds of generating posterior beliefs that just meet the threshold. The firm's optimal design in such a setting can be viewed as a liberal or aggressive set of accounting policies. Although our model is highly stylized to focus on but one tension the firm faces in choosing the properties of its reporting system, we believe that meeting crucial thresholds could be an overriding concern for some firms during time spans long enough to influence financial accounting policy decisions. The flexibility afforded firms by accounting standards in choosing accounting policies constitutes a natural device for firms to employ in seeking to meet thresholds or otherwise influence the beliefs held by financial statement users.

The prospect of receiving private information, over which disclosure by the firm is discretionary, induces the firm to change the properties of its financial reporting system. When private signals are less informative, the firm directs its financial reporting system toward providing more informative favorable reports. This is because such reports have to raise the posterior beliefs sufficiently to offset the negative influence of the potential non-disclosure of a private signal, given that such non-disclosure may be due to an unfavorable private signal or no private signal having been received. When private signals are more informative and prior beliefs are pessimistic, the firm chooses less informative favorable reports, anticipating that disclosure of a sufficiently favorable private signal would compensate for the effect of an unfavorable report on posterior expectations. The firm's financial reporting system choices in the remaining case of more informative private signals and optimistic prior beliefs are more complex involving both less informative favorable reports and more informative unfavorable reports. Constructively, financial reports and private signals are partial substitutes. Less informative private signals in general imply a choice of more informative financial reporting systems.

Broadly speaking, in settings where outside parties employ only a threshold decision rule, discretionary disclosure of private information provides no benefit to the firm beyond that achievable through a judicious choice of a public financial reporting system. The result that the sender (firm) is at least weakly better off sans private information is generalizable to a large class of distributional and payoff assumptions. While likely to be less descriptive of situations that firms may face, absent a marginal further benefit to disclosing favorable private signals *per se*, an alternative equilibrium exists in which the firm does not disclose even those signals which would increase the likelihood of meeting or exceeding the threshold. Comparing regimes with discretionary and mandatory disclosure of private information, there are conditions under which the firm may prefer one or the other. In particular, a combination of optimistic prior beliefs and highly informative private signals implies a preference for mandatory disclosure. Although useful as a theoretical benchmark, implementing mandatory disclosure would require monitoring of the receipt of private information and penalties for non-compliance, which may be infeasible or, at best, very costly.

While we have focused on the application of our model to financial reporting by firms, the structure we employ may also be suitable for characterizing reporting choices for intermediaries that gather information for distribution to other parties. Financial analysts may curry favor with firms by seeking to primarily acquire good news in formulating their estimates and ratings provided to investors, which may or may not be reinforced by information subsequently received. Information gathering is one of the tasks that sell-side analysts perform (Michaely and Womack 2005). To the extent that this task may be biased is consistent with the *ex ante* concept of information system choices in our model. Bond rating agencies have likewise been thought to limit the extent to which they search for news that might result in lower ratings or downgrades. Jiang, Stanford, and Xie (2012) find evidence of upwardly biased ratings for issuer-pay firms. Whether bias manifests in information gathering or later in the process is an open question. Extending the jurisprudence context of Kamenica and Gentzkow (2011), a prosecutor who slants information gathered for a fair-minded judge may subsequently be faced with a decision regarding whether to suppress or reveal information subsequently obtained. The advocacy nature of the legal system suggests a high likelihood of misaligned preferences, and rules of discovery and penalties for evidence tampering suggest incentives for biases to enter at the information gathering stage. While withholding subsequently obtained evidence is unlawful, recent high-profile cases suggest it still occurs (Patrice 2015; Simmerman 2012).

Giving some thought to empirical applications, we note that in 2005 the S.E.C. liberalized its "quiet period" policies to allow more information to be communicated for certain organizations following the filing of a registration statement. For IPOs this period is often referred to as a "cooling-off period." In the context of our study, such a period, if enforced, may serve as a commitment device that benefits the firm, notwithstanding that its effect may be to diminish the informativeness of prospectuses. Relaxing these policies may have the opposite effect suggesting a natural experiment to test our predictions may be feasible. There is some prospect that these policies may be further liberalized or even eliminated given the commonly held view echoed by Fortune magazine's 2011 feature article, "It's time to kill the IPO quiet period." Given that the ability and motivation to meet a crucial threshold may only be present and substantial for some firms, there is scope for cross-sectional differences that could contribute to the power of one's tests.

APPENDIX

Proof of Lemma 1: The firm discloses $s = h$ whenever:

$$\begin{aligned}
\Delta_h &\equiv E[\theta|r, m = h] - E[\theta|r, m = \emptyset] \\
&= \Pr(\theta = H|r, m = h) - \Pr(\theta = H|r, m = \emptyset) \\
&\geq 0, \quad \forall r = g, b.
\end{aligned}$$

It is straightforward to verify that, because $\gamma_H > \gamma_L$ by assumption, $\Pr(\theta = H|r, m = h) \geq \Pr(\theta = H|r, m = \emptyset)$, $\forall r = g, b$. Hence, the firm discloses $s = h$. Next, we show that the firm withholds $s = l$:

$$\begin{aligned}
\Delta_l &\equiv E[\theta|r, m = l] - E[\theta|r, m = \emptyset] \\
&= \Pr(\theta = H|r, m = l) - \Pr(\theta = H|r, m = \emptyset) \\
&\leq 0, \quad \forall r = g, b.
\end{aligned}$$

It is straightforward to verify that, because $\gamma_H > \gamma_L$ by assumption, $\Pr(\theta = H|r, m = l) \leq \Pr(\theta = H|r, m = \emptyset)$, $\forall r = g, b$. Hence, the firm withholds $s = l$. ■

Proof of Proposition 1: $\mathcal{P}1(D)$ can be rewritten as:

$$\begin{aligned}
&\max_{\beta_H, \beta_L} \alpha\beta_H + (1-\alpha)\beta_L + q(\alpha(1-\beta_H)\gamma_H + (1-\alpha)(1-\beta_L)\gamma_L) \\
&\text{s.t.} \quad E[\theta|r = g, m = h] \geq k, \\
&\qquad\quad E[\theta|r = g, m = \emptyset] \geq k, \\
&\qquad\quad E[\theta|r = b, m = h] \geq k, \\
&\qquad\quad 1 \geq \beta_H \geq \beta_L \geq 0.
\end{aligned}$$

The first condition is slack whenever either the second, or the third condition are satisfied (because $E[\theta|r = g, m = h] \geq E[\theta|r = g, m = \emptyset]$ and $E[\theta|r = g, m = h] \geq E[\theta|r = b, m = h]$). Therefore, the Lagrangian is

$$\begin{aligned}
\mathcal{L}_1 &= \alpha\beta_H + (1-\alpha)\beta_L + q(\alpha(1-\beta_H)\gamma_H + (1-\alpha)(1-\beta_L)\gamma_L) \\
&\quad + \mu_1(E[\theta|r = g, m = \emptyset] - k) + \mu_2(E[\theta|r = b, m = h] - k) \\
&\quad + \mu_3(1-\beta_H) + \mu_4(\beta_H - \beta_L) + \mu_5\beta_L.
\end{aligned}$$

The Karush-Kuhn-Tucker stationarity conditions are:

$$\begin{aligned}
\frac{\partial \mathcal{L}_1}{\partial \beta_H} &= \alpha(1-q\gamma_H) \\
&\quad +\mu_1 \frac{\partial E[\theta|r=g,m=\emptyset]}{\partial \beta_H} + \mu_2 \frac{\partial E[\theta|r=b,m=h]}{\partial \beta_H} \\
&\quad -\mu_3+\mu_4=0 \qquad (1) \\
\frac{\partial \mathcal{L}_1}{\partial \beta_L} &= (1-\alpha)(1-q\gamma_L) \\
&\quad +\mu_1 \frac{\partial E[\theta|r=g,m=\emptyset]}{\partial \beta_L} + \mu_2 \frac{\partial E[\theta|r=b,m=h]}{\partial \beta_L} \qquad (2) \\
&\quad -\mu_4+\mu_5=0,
\end{aligned}$$

the Karush-Kuhn-Tucker feasibility conditions are:

$$\begin{aligned}
E[\theta|r=g,m=\emptyset]-k &\geq 0 \qquad (3) \\
E[\theta|r=b,m=h]-k &\geq 0 \qquad (4) \\
1-\beta_H &\geq 0 \qquad (5) \\
\beta_H-\beta_L &\geq 0, \qquad (6) \\
\beta_L &\geq 0 \qquad (7) \\
\mu_i &\geq 0, \quad i=1,2,3,4,5. \qquad (8)
\end{aligned}$$

and the Karush-Kuhn-Tucker complementarity slackness conditions are:

$$\begin{aligned}
\mu_1(E[\theta|r=g,m=\emptyset]-k) &= 0 \qquad (9) \\
\mu_2(E[\theta|r=b,m=h]-k) &= 0 \qquad (10) \\
\mu_3(1-\beta_H) &= 0 \qquad (11) \\
\mu_4(\beta_H-\beta_L) &= 0 \qquad (12) \\
\mu_5\beta_L &= 0. \qquad (13)
\end{aligned}$$

With five complementarity slackness conditions there are $2^5 = 32$ cases. We can immediately rule out:

(i) All cases with $\mu_3 > 0$ (so $\beta_H = 1$) because if $\beta_H = 1$, then $E[\theta|r = b, m = \emptyset] = 0 < k$ which is a contradiction;

(ii) All cases with $\mu_1 > 0$, $\mu_5 > 0$ (so $E[\theta|r = g, m = \emptyset] = k$, $\beta_L = 0$) because if $\beta_L = 0$, then $E[\theta|r = g, m = \emptyset] = 1 > k$ which is a contradiction;

(iii) All cases with $\mu_4 > 0$ ($\beta_H = \beta_L$), because if $\beta_H = \beta_L$, then $E[\theta|r = g, m = \emptyset] = E[\theta|r = b, m = \emptyset] < k$ which is a contradiction.

(iv) All cases with $\mu_2 = 0$, $\mu_3 = 0$ (so $E[\theta|r = b, m = h] > k$ and $1 > \beta_H$), because then $\frac{\partial \mathcal{L}_1}{\partial \beta_H} = \alpha(1 - q\gamma_H) + \mu_1 \frac{\partial E[\theta|r=g,m=\emptyset]}{\partial \beta_H} + \mu_4 > \mu_1 \frac{\partial E[\theta|r=g,m=\emptyset]}{\partial \beta_H} \geq 0$, implying $\beta_H = 1$ which is a contradiction;

(D) All cases with $\mu_1 = 0$, $\mu_4 = 0$ (so $E[\theta|r = g, m = \emptyset] > k$ and $\beta_H > \beta_L$), because then $\frac{\partial \mathcal{L}_1}{\partial \beta_L} = (1 - \alpha)(1 - q\gamma_L) + \mu_2 \frac{\partial E[\theta|r=b,m=h]}{\partial \beta_L} + \mu_5 > \mu_2 \frac{\partial E[\theta|r=b,m=h]}{\partial \beta_L} \geq 0$, implying $\beta_L = 1 \geq \beta_H$ which is a contradiction;

We are left with only one case to consider:

Case 1: $\mu_1 > 0$, $\mu_2 > 0$, $\mu_3 = 0$, $\mu_4 = 0$, $\mu_5 = 0$, $E[\theta|r = g, m = \emptyset] = k$, $E[\theta|r = b, m = h] = k$, $1 > \beta_H > \beta_L > 0$

We solve the four equations below

$$E[\theta|r = g, m = \emptyset] - k = 0$$
$$E[\theta|r = b, m = h] - k = 0$$
$$\alpha(1 - q\gamma_H) + \mu_1 \frac{\partial E[\theta|r = g, m = \emptyset]}{\partial \beta_H} + \mu_2 \frac{\partial E[\theta|r = b, m = h]}{\partial \beta_H} = 0$$
$$(1 - \alpha)(1 - q\gamma_L) + \mu_1 \frac{\partial E[\theta|r = g, m = \emptyset]}{\partial \beta_L} + \mu_2 \frac{\partial E[\theta|r = b, m = h]}{\partial \beta_L} = 0$$

and get

$$\beta_H = \frac{(\gamma_H \alpha(1 - k) - \gamma_L k(1 - \alpha))(1 - q\gamma_L)}{\alpha(\gamma_H - \gamma_L)(1 - k)}$$
$$\beta_L = \frac{(\gamma_H \alpha(1 - k) - \gamma_L k(1 - \alpha))(1 - q\gamma_H)}{k(\gamma_H - \gamma_L)(1 - \alpha)}$$
$$\mu_1 = \frac{\Gamma(\gamma_H(1 - k)\alpha - \gamma_L k(1 - \alpha))(\gamma_L k + \gamma_H(1 - k - \gamma_L q))}{(\gamma_H - \gamma_L)^2(1 - k)^2 k^2}$$
$$\mu_2 = \frac{\Gamma\gamma_H\gamma_L(k - \alpha(1 - q(\gamma_H k + \gamma_L(1 - k)) - \gamma_L kq))}{(\gamma_H - \gamma_L)^2(1 - k)^2 k^2}$$

where $\Gamma \equiv (1-\gamma_H q)(1-\gamma_L q)$. It is straightforward to verify that this case is feasible in a sense that $1 > \beta_H > \beta_L > 0$ and $\mu_1 > 0$, $\mu_2 > 0$ whenever $\alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H (1-k)}$. For future reference,

– if $\alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H (1-k)}$, then

$$\begin{aligned} 1 &> \frac{(\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha))(1-q\gamma_L)}{\alpha(\gamma_H - \gamma_L)(1-k)} \\ &= \beta_H \\ &> \beta_L \\ &= \frac{(\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha))(1-q\gamma_H)}{k(\gamma_H - \gamma_L)(1-\alpha)} \\ &> 0 \end{aligned}$$

and the value of $\mathcal{P}1(D)$ is:

$$\frac{M_A \cdot M_B + M_C}{k(1-k)(\gamma_H - \gamma_L)};$$

where $M_A \equiv \alpha(1-(\gamma_H(1-k)+\gamma_L k)q)$, $M_B \equiv -(\gamma_L k + \gamma_H(1-k-\gamma_L q))$ and $M_C \equiv \gamma_L k(1-q(\gamma_L k + \gamma_H(2-k-\gamma_L q)))$.

– if $\alpha < \frac{\gamma_L k}{\gamma_L k + \gamma_H (1-k)}$, then $\mathcal{P}1(D)$ is not feasible.

$\mathcal{P}2(D)$ can be rewritten as:

$$\begin{aligned} &\max_{\beta_H, \beta_L} q(\alpha\gamma_H + (1-\alpha)\gamma_L) \\ &\text{s.t.} \quad E[\theta | r = g, m = h] \geq k, \\ &\qquad E[\theta | r = b, m = h] \geq k, \\ &\qquad 1 \geq \beta_H \geq \beta_L \geq 0. \end{aligned}$$

The maximand is independent of β_H and β_L so we just need to ensure that the conditions are satisfied. The first condition is slack if the second condition holds (because $E[\theta | r = g, m = h] > E[\theta | r = b, m = h]$) so we only need to verify that the second and third condition are feasible. Substituting for $E[\theta | r = b, m = h]$ in the second condition and rearranging we get

$$\frac{1-\beta_H}{1-\beta_L} \geq \frac{\gamma_L}{\gamma_H} \frac{(1-\alpha)}{(1-k)} \frac{k}{\alpha}; \tag{14}$$

$$1 \geq \beta_H \geq \beta_L \geq 0. \tag{15}$$

We note that if (15) is satisfied, then $\frac{1-\beta_H}{1-\beta_L} \in [0,1]$. Therefore:

(i) if the RHS of (14), is bigger than one, i.e., when

$$\frac{\gamma_L}{\gamma_H}\frac{(1-\alpha)}{(1-k)}\frac{k}{\alpha} \geq 1 \quad \Leftrightarrow \quad \alpha \leq \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$$

then (14) cannot be satisfied for any β_H and β_L that satisfy (15).

(ii) if the RHS of (14), is smaller than one, i.e., when

$$\frac{\gamma_L}{\gamma_H}\frac{(1-\alpha)}{(1-k)}\frac{k}{\alpha} < 1 \quad \Leftrightarrow \quad \alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$$

then the firm sets β_H and β_L that satisfy (14) and (15) simultaneously.

For future reference,

– if $\alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$, then the value of $\mathcal{P}2(D)$ is $q(\alpha\gamma_H + (1-\alpha)\gamma_L)$

– if $\alpha \leq \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$ then $\mathcal{P}2(D)$ is not feasible.

$\mathcal{P}3(D)$ can be rewritten as:

$$\begin{aligned} &\max_{\beta_H,\beta_L} \alpha\beta_H + (1-\alpha)\beta_L \\ &\text{s.t.} \quad E[\theta|r=g, m=h] \geq k, \\ &\qquad\;\; E[\theta|r=g, m=\emptyset] \geq k, \\ &\qquad\;\; 1 \geq \beta_H \geq \beta_L \geq 0. \end{aligned}$$

Setting the second condition binding ensures that the first condition is satisfied (because $E[\theta|r=g, m=h] \geq E[\theta|r=g, m=\emptyset]$) and allows us to express β_L:

$$E[\theta|r=g, m=\emptyset] = k \quad \Rightarrow \quad \beta_L = \beta_H \frac{\alpha(1-k)(1-\gamma_H q)}{k(1-\alpha)(1-\gamma_L q)}$$

Substituting and simplifying, we can rewrite the optimization program as:

$$\begin{aligned} &\max_{\beta_H,\beta_L} \alpha\beta_H \left(1 + \frac{(1-k)(1-\gamma_H q))}{k(1-\gamma_L q)}\right) \\ &\text{s.t.} \quad 1 \geq \beta_H \geq \beta_L \geq 0 \end{aligned}$$

Taking the derivative with respect to β_H yields

$$\alpha\left(1 + \frac{(1-k)(1-\gamma_H q)}{k(1-\gamma_L q)}\right) > 0$$

and therefore $\beta_H = 1$ and $\beta_L = \frac{\alpha(1-k)(1-\gamma_H q)}{k(1-\alpha)(1-\gamma_L q)}$ (note that $1 \geq \beta_H \geq \beta_L \geq 0$ is satisfied because $0 < \alpha < k < 1$ and $0 \leq \gamma_L \leq \gamma_H \leq 1$ by assumption). For future reference, the value of $\mathcal{P}3(D)$ is $\alpha\left(1 + \frac{(1-k)(1-\gamma_H q)}{k(1-\gamma_L q)}\right)$.

$\mathcal{P}4(D)$ can be rewritten as:

$$\begin{aligned}
&\max_{\beta_H,\beta_L} q(\alpha\beta_H\gamma_H + (1-\alpha)\beta_L\gamma_L) \\
&\text{s.t.} \quad E[\theta|r=g, m=h] \geq k, \\
&\qquad\quad 1 \geq \beta_H \geq \beta_L \geq 0.
\end{aligned}$$

The Lagrangian is:

$$\begin{aligned}
\mathcal{L}_4 \;=\; & q(\alpha\beta_H\gamma_H + (1-\alpha)\beta_L\gamma_L) \\
& +\mu_1(E[\theta|r=g, m=h] - k) + \mu_2(1-\beta_H) + \mu_3(\beta_H - \beta_L) + \mu_4\beta_L.
\end{aligned}$$

The Karush-Kuhn-Tucker stationarity conditions are

$$\frac{\partial\mathcal{L}_4}{\partial\beta_H} = q\alpha\gamma_H + \mu_1\frac{\partial E[\theta|r=g, m=h]}{\partial\beta_H} - \mu_2 + \mu_3 = 0 \tag{16}$$

$$\frac{\partial\mathcal{L}_4}{\partial\beta_L} = q(1-\alpha)\gamma_L + \mu_1\frac{\partial E[\theta|r=g, m=h]}{\partial\beta_L} - \mu_3 + \mu_4 = 0, \tag{17}$$

the Karush-Kuhn-Tucker feasibility conditions are:

$$E[\theta|r=g, m=h] - k \geq 0 \tag{18}$$

$$1-\beta_H \geq 0 \tag{19}$$

$$\beta_H - \beta_L \geq 0 \tag{20}$$

$$\beta_L \geq 0, \tag{21}$$

$$\mu_i \geq 0, \quad i = 1,2,3,4. \tag{22}$$

and the Karush-Kuhn-Tucker complementarity slackness conditions are:

$$\mu_1(E[\theta|r=g, m=h] - k) = 0 \tag{23}$$

$$\mu_2(1-\beta_H) = 0 \tag{24}$$

$$\mu_3(\beta_H - \beta_L) = 0 \tag{25}$$

$$\mu_4\beta_L = 0. \tag{26}$$

With four complementarity slackness conditions there are $2^4 = 16$ cases. We can immediately rule out:

(i) All cases with $\mu_3 > 0$, $\mu_4 > 0$ (so $\beta_H = \beta_L = 0$), because then $E[\theta|r = g, m = h] = 0 < k$, which is a contradiction;

(ii) All cases with $\mu_1 > 0$, $\mu_4 > 0$ (so $E[\theta|r = g, m = h] = k$, $\beta_L = 0$) because if $\beta_L = 0$, then $E[\theta|r = g, m = h] = 1 > k$ which is a contradiction;

(iii) All cases with $\mu_2 = 0$ (so $1 > \beta_H$), because then $\frac{\partial \mathcal{L}_4}{\partial \beta_H} = q\alpha\gamma_H + \mu_1 \frac{\partial E[\theta|r=g,m=h]}{\partial \beta_H} + \mu_3 \geq \mu_1 \frac{\partial E[\theta|r=g,m=h]}{\partial \beta_H} \geq 0$, implying $\beta_H = 1$ which is a contradiction;

(iv) All cases with $\mu_1 = 0$ and $\mu_3 = 0$ (so $E[\theta|r = g, m = h] > k$ and $1 \geq \beta_H > \beta_L$) because $\frac{\partial \mathcal{L}_4}{\partial \beta_L} = q(1-\alpha)\gamma_L + \mu_4 \geq 0$, implying $\beta_L = 1$ which is a contradiction;

We are left with only three cases to consider:

Case 1: $\mu_1 > 0$, $\mu_2 > 0$, $\mu_3 = 0$, $\mu_4 = 0$, $E[\theta|r = g, m = h] = k$, $1 = \beta_H > \beta_L > 0$

$$E[\theta|r = g, m = h] = k \quad \Rightarrow \quad \beta_L = \beta_H \frac{\alpha(1-k)\gamma_H}{k(1-\alpha)\gamma_L}$$

Substituting $\beta_H = 1$ and $\beta_L = \frac{\alpha(1-k)\gamma_H}{k(1-\alpha)\gamma_L}$ into (17) and solving yields $\mu_1 = \frac{\alpha\gamma_H q}{k^2} > 0$. Substituting β_H, β_L and μ_1 into (17) yields $\mu_2 = \frac{\alpha\gamma_H q}{k} > 0$. This case is feasible only if $\beta_L = \frac{\alpha(1-k)\gamma_H}{k(1-\alpha)\gamma_L} < \beta_H = 1$, which is equivalent to the requirement $\alpha < \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$.

Case 2: $\mu_1 > 0$, $\mu_2 > 0$, $\mu_3 > 0$, $\mu_4 = 0$, $E[\theta|r = g, m = h] = k$, $1 = \beta_H = \beta_L > 0$

If $1 = \beta_H = \beta_L$, then $E[\theta|r = g, m = h] = \frac{\alpha\gamma_H}{\alpha\gamma_H + (1-\alpha)\gamma_H}$, i.e., the investors rationally ignore the report because it is uninformative. By $E[\theta|r = g, m = h] = k$ it follows that this case can only be feasible when

$$\frac{\gamma_L}{\gamma_H} = \frac{\alpha(1-k)}{k(1-\alpha)} \tag{27}$$

Substituting $1 = \beta_H = \beta_L$ and (27) into (17) yields $\mu_1 = \frac{\alpha\gamma_H q(1-k) - k\mu_3}{(1-k)k^2}$. Substituting $1 = \beta_H = \beta_L$, (27) and μ_1 into (16) yields $\mu_2 = \frac{\alpha\gamma_H q}{k} > 0$. Substituting $1 = \beta_H = \beta_L$, μ_1 and μ_2 into (16) yields $\mu_3 = \frac{\alpha\gamma_H(1-k)q}{k} > 0$. Then, $\mu_1 = 0$ which is a contradiction.

Case 3: $\mu_1 = 0$, $\mu_2 > 0$, $\mu_3 > 0$, $\mu_4 = 0$, $E[\theta|r = g, m = h] > k$, $1 = \beta_H = \beta_L > 0$

Substituting $\beta_H = \beta_L = 1$ into (17) implies that $\mu_3 = q(1-\alpha)\gamma_L > 0$. Substituting $\beta_H = \beta_L = 1$ and μ_3 into (16) implies $\mu_2 = q(\alpha\gamma_H + (1-\alpha)\gamma_L) > 0$. This case is feasible if $E[\theta|r=g,m=h]=\frac{\alpha\beta_H\gamma_H}{\alpha\beta_H\gamma_H+(1-\alpha)\beta_L\gamma_H}=\frac{\alpha\gamma_H}{\alpha\gamma_H+(1-\alpha)\gamma_H}>k$ which is equivalent to the requirement $\alpha>\frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$. However, we note that if $\beta_H=\beta_L=1$ the investors rationally ignore the report because it is uninformative (this case is considered under a separate optimization program). Hence, this solution reduces to the solution to $\mathcal{P}2(D)$.

For future reference,

- if $\alpha>\frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$, then the solutions to $\mathcal{P}4(D)$ and $\mathcal{P}2(D)$ are equivalent.
- if $\alpha<\frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$, then $1=\beta_H>\beta_L=\frac{\alpha(1-k)\gamma_H}{k(1-\alpha)\gamma_L}>0$ and the value of $\mathcal{P}4(D)$ is $\frac{q\alpha\gamma_H}{k}$.

Below is a summary of the values of the programs:

(A) If $\alpha<\frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$, then

  (a) $\mathcal{P}1(D)$ is not feasible;

  (b) $\mathcal{P}2(D)$ is not feasible;

  (c) The value of $\mathcal{P}3(D)$ is $\alpha\left(1+\frac{(1-k)(1-\gamma_H q)}{k(1-\gamma_L q)}\right)$;

  (d) The value of $\mathcal{P}4(D)$ is $\frac{q\alpha\gamma_H}{k}$.

(B) If $\alpha>\frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$, then

  (a) The value of $\mathcal{P}1(D)$ is: and the value of $\mathcal{P}1(D)$ is:

$$\frac{M_A\cdot M_B+M_C}{k(1-k)(\gamma_H-\gamma_L)};$$

  where $M_A\equiv\alpha(1-(\gamma_H(1-k)+\gamma_L k)q)$, $M_B\equiv-(\gamma_L k+\gamma_H(1-k-\gamma_L q))$ and $M_C\equiv\gamma_L k(1-q(\gamma_L k+\gamma_H(2-k-\gamma_L q)))$.

  (b) The value of $\mathcal{P}2(D)$ is $q(\alpha\gamma_H+(1-\alpha)\gamma_L)$;

  (c) The value of $\mathcal{P}3(D)$ is $\alpha\left(1+\frac{(1-k)(1-\gamma_H q)}{k(1-\gamma_L q)}\right)$;

  (d) The value of $\mathcal{P}4(D)$ is equal to the value of $\mathcal{P}2(D)$, $q(\alpha\gamma_H+(1-\alpha)\gamma_L)$.

As a last step we compare the values of the programs. The comparison reveals that if $\gamma_H<\overline{g}$, then $\mathcal{P}3(D)$ has the highest value (for any $\alpha\in(0,1)$). Hence, if Condition 1 is satisfied the firm sets $\beta_H^*=1$ and $\beta_L^*=\frac{\alpha(1-k)(1-\gamma_H q)}{k(1-\alpha)(1-\gamma_L q)}$ and the threshold is met or exceeded whenever the public report is favorable. Note that $\beta_H^*=1>\beta_L^*>0$. ■

Proof of Proposition 2:

Item (i): Using the proof of Proposition 1, we note that if $\alpha < \frac{\gamma_L k}{\gamma_L k + \gamma_H (1-k)}$ and $\gamma_H > \overline{g}$, then $\mathcal{P}4(D)$ has the highest value. The firm sets $\beta_H^{**} = 1$ and $\beta_L^{**} = \frac{\alpha(1-k)\gamma_H}{k(1-\alpha)\gamma_L} \in (0,1)$ and the threshold k is met whenever both the public report and the disclosure are favorable. As a last step we verify that this case is feasible, i.e., that $\overline{g} < 1$. This is true when $q > \overline{q} \equiv \frac{2-k(1-\gamma_L)-\sqrt{(1-\gamma_L)(4-k(4-k(1-\gamma_L))}}{2\gamma_L}$ (note that $\overline{q} < 1$).

Item (ii): Using the proof of Proposition 1, we note that if $\alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H (1-k)}$ and $\gamma_H > \overline{g}$, then $\mathcal{P}1(D)$ has the highest value. The firm sets

$$\begin{aligned}
1 &> \frac{(\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha))(1-q\gamma_L)}{\alpha(\gamma_H - \gamma_L)(1-k)} \\
&= \beta_H^{***} \\
&> \beta_L^{***} \\
&= \frac{(\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha))(1-q\gamma_H)}{k(\gamma_H - \gamma_L)(1-\alpha)} \\
&> 0
\end{aligned}$$

and the threshold is met or exceeded whenever either the public report or the disclosure are favorable. Using the proof of item (i), we note that this case is feasible. ■

Proof of Footnote 16 claim: The variance of posterior expectations (VPE), conditioning on the report, r, is defined as a function of the $\boldsymbol{\beta}$ vector as

$$\text{VPE}(\boldsymbol{\beta}) \equiv Var\left[E\left[\theta|r\right]\right] = E[(E\left[\theta|r\right] - E\left[E\left[\theta|r\right]\right])^2],$$

which is equal to $E[(\Pr\left[\theta = 1|r\right] - \alpha)^2]$ and can be expressed as

$$\text{VPE}(\boldsymbol{\beta}) = \frac{(\alpha-1)^2 \ \alpha^2 \left(\beta_H - \beta_L\right)^2}{\left(1 - \alpha\beta_H - (1-\alpha)\beta_L\right) \left(\alpha\beta_H + (1-\alpha)\beta_L\right)}.$$

Plugging in the values for $\boldsymbol{\beta}^*$ and $\boldsymbol{\beta}^{***}$ yields

$$\text{VPE}(\boldsymbol{\beta}^*) = \frac{\alpha \ (\alpha \ + k \left(\alpha\gamma_H q + \gamma_L \left(1-\alpha\right) q - 1\right) - \alpha\gamma_H q)}{q(\gamma_H \left(1-k\right) + \gamma_L k) - 1},$$

and

$$\text{VPE}(\boldsymbol{\beta}^{***}) = \frac{\alpha\gamma_H \left(1-k\right) - k\gamma_L \left(1-\alpha\right)}{\alpha(\gamma_H \left(1-k\right) + \gamma_L k)} * VCE\left(\boldsymbol{\beta}^*\right).$$

For feasible values of the exogenous parameters, i.e., $0 < \alpha < k < 1$ and $0 < \gamma_L \leq \gamma_H < 1$, we have $\text{VPE}(\boldsymbol{\beta}^*) > \text{VPE}(\boldsymbol{\beta}^{***})$. If $\gamma_L = 0$, then $\text{VPE}(\boldsymbol{\beta}^*) = \text{VPE}(\boldsymbol{\beta}^{***})$. ■

Proof of Proposition 4:

$\mathcal{P}1(M)$ is identical to $\mathcal{P}4(D)$ from the proof of Proposition 1.

$\mathcal{P}2(M)$ can be rewritten as:

$$
\begin{aligned}
&\max_{\boldsymbol{\beta}} q\left(\alpha\beta_H\gamma_H + (1-\alpha)\,\beta_L\gamma_L\right) + p\left(\alpha\beta_H + (1-\alpha)\,\beta_L\right)\\
&\text{s.t.}\quad E[\theta|r=g, m=h] \geq k,\\
&\qquad\ \ E[\theta|r=g, m=\emptyset] \geq k,\\
&\qquad\ \ 1 \geq \beta_H \geq \beta_L \geq 0;
\end{aligned}
$$

The first constraint is slack if the second constraint holds. The second constraint binds:

$$\frac{\beta_H\alpha\,(1-k)}{k\,(1-\alpha)} = \beta_L$$

and so the optimization program can be rewritten as

$$\max_{\boldsymbol{\beta}} \beta_H\left(q\left(\alpha\gamma_H + (1-\alpha)\,\frac{\alpha\,(1-k)}{k\,(1-\alpha)}\gamma_L\right) + (1-q)\left(\alpha + (1-\alpha)\,\frac{\alpha\,(1-k)}{k\,(1-\alpha)}\right)\right)$$

We note that the expected payoff is increasing in β_H and therefore $\beta_H = 1$. Substituting, we find that $\beta_L = \frac{\alpha(1-k)}{(1-\alpha)k} < 1$. For future reference the value of the optimization program is $\frac{\alpha}{k}\left(q\left(k\gamma_H + (1-k)\,\gamma_L\right) + (1-q)\right)$.

$\mathcal{P}3(M)$ is identical to $\mathcal{P}2(D)$ from the proof of Proposition 1.

$\mathcal{P}4(M)$ can be rewritten as:

$$
\begin{aligned}
&\max_{\boldsymbol{\beta}} (1-q)\left(\beta_L\,(1-\alpha) + \alpha\beta_H\right)\\
&\text{s.t.}\quad E[\theta|r=g, m=h] \geq k,\\
&\qquad\ \ E[\theta|r=b, m=h] \geq k,\\
&\qquad\ \ E[\theta|r=g, m=\emptyset] \geq k,\\
&\qquad\ \ 1 \geq \beta_H \geq \beta_L \geq 0;
\end{aligned}
$$

The first constraint is slack if the third constraint holds. The Lagrangean is:

$$
\begin{aligned}
\mathcal{L}_4 \;=\;& (1-q)\left(\beta_L\,(1-\alpha) + \alpha\beta_H\right) + \mu_1\left(E[\theta|r=b, m=h] - k\right)\\
&+\mu_2\left(E[\theta|r=g, m=\emptyset] - k\right) + \mu_3\left(1-\beta_H\right) + \mu_4\left(\beta_H - \beta_L\right) + \mu_5\left(\beta_L\right)
\end{aligned}
$$

The Karush-Kuhn-Tucker stationarity conditions are

$$\begin{aligned}
\frac{\partial \mathcal{L}_4}{\partial \beta_H} &= (1-q)\alpha + \mu_1 \frac{\partial E[\theta|r=b, m=h]}{\partial \beta_H} + \mu_2 \frac{\partial E[\theta|r=g, m=\emptyset]}{\partial \beta_H} - \mu_3 + \mu_4 = 0 \\
\frac{\partial \mathcal{L}_4}{\partial \beta_L} &= (1-q)(1-\alpha) + \mu_1 \frac{\partial E[\theta|r=b, m=h]}{\partial \beta_L} + \mu_2 \frac{\partial E[\theta|r=g, m=\emptyset]}{\partial \beta_L} - \mu_4 + \mu_5 = 0,
\end{aligned}$$

the Karush-Kuhn-Tucker feasibility conditions are

$$\begin{aligned}
\frac{\alpha(1-\beta_H)\gamma_H}{\alpha(1-\beta_H)\gamma_H + (1-\alpha)(1-\beta_L)\gamma_L} - k &\geq 0 \\
\frac{\alpha\beta_H}{\alpha\beta_H + (1-\alpha)\beta_L} - k &\geq 0 \\
1 - \beta_H &\geq 0 \\
\beta_H - \beta_L &\geq 0 \\
\beta_L &\geq 0
\end{aligned}$$

and the Karush-Kuhn-Tucker complementarity slackness conditions are

$$\begin{aligned}
\left(\frac{\alpha(1-\beta_H)\gamma_H}{\alpha(1-\beta_H)\gamma_H + (1-\alpha)(1-\beta_L)\gamma_L} - k\right)\mu_1 &= 0 \\
\left(\frac{\alpha\beta_H}{\alpha\beta_H + (1-\alpha)\beta_L} - k\right)\mu_2 &= 0 \\
(1-\beta_H)\mu_3 &= 0 \\
(\beta_H - \beta_L)\mu_4 &= 0 \\
\beta_L\mu_5 &= 0
\end{aligned}$$

We know:

1. $\beta_H > \beta_L$ because, otherwise, if $\Pr(H|r=g, m=\emptyset) - k \geq 0$ then it has to be that $\Pr(H|r=b, m=\emptyset) - k \geq 0$. But we know that $\Pr(H|r=b, m=\emptyset) < \alpha \Rightarrow \Pr(H|r=b, m=\emptyset) - k < \alpha - k < 0$. It follows that $\mu_4 = 0$.

2. $\beta_H < 1$, because otherwise $\Pr(H|r=b, m=h) = 0 \Rightarrow \Pr(H|r=b, m=h) - k < 0$, so it follows that $\mu_3 = 0$.

3. Since $\mu_4 = 0$, it must be true that $\mu_2 > 0$ because otherwise $\frac{\partial \mathcal{L}_4}{\partial \beta_L} > 0$ implying $\beta_L = 1$ which contradicts $\beta_L < \beta_H < 1$.

4. Since $\mu_3 = 0$ ($\beta_H < 1$), it must be true that $\mu_1 > 0$ because otherwise $\frac{\partial \mathcal{L}_4}{\partial \beta_H} > 0$ implying $\beta_H = 1$ which is a contradiction.

It follows that β_H and β_L are defined by the binding constraints:

$$\begin{aligned}
\beta_H &= \frac{\gamma_H \alpha (1-k) - \gamma_L k (1-\alpha)}{\alpha (1-k)(\gamma_H - \gamma_L)} < 1 \\
\beta_L &= \frac{\gamma_H \alpha (1-k) - \gamma_L k (1-\alpha)}{k (1-\alpha)(\gamma_H - \gamma_L)} < \beta_H
\end{aligned}$$

If $\gamma_H \alpha (1-k) - \gamma_L k (1-\alpha) < 0$, then $\beta_L = 0$ and the first constraint gives us

$$\frac{\gamma_H \alpha (1-k) - \gamma_L k (1-\alpha)}{\gamma_H \alpha (1-k)} = \beta_H$$

but because we assumed $\gamma_H \alpha (1-k) - \gamma_L k (1-\alpha) < 0$, this implies $\beta_H < 0$, which is not feasible. The second constraint gives us $1 - k = 0$, contradicts our assumption of $1 > k$. So $\mathcal{P}4(M)$ has a solution only for $\gamma_H > \frac{k(1-\alpha)}{\alpha(1-k)} \gamma_L$. For future reference, the value of the optimization program is $q(\alpha \gamma_H + \gamma_L (1-\alpha)) + (1-q) \frac{\alpha(1-k)\gamma_H - k(1-\alpha)\gamma_L}{(1-k)(\gamma_H - \gamma_L)k}$.

$\mathcal{P}5(M)$ can be rewritten as:

$$\begin{aligned}
&\max_{\beta} \alpha \beta_H + (1-\alpha) \beta_L \\
&\text{s.t.} \quad E[\theta | r = g, m = h] \geq k, \\
&\qquad\ \ E[\theta | r = g, m = \emptyset] \geq k, \\
&\qquad\ \ E[\theta | r = g, m = l] \geq k, \\
&\qquad\ \ 1 \geq \beta_H \geq \beta_L \geq 0
\end{aligned}$$

The first and second constraints are slack if the third constraint is satisfied. The expected payoff is increasing in both β_H and β_L. We examine the third constraint and note that:

$$\frac{\partial}{\partial \beta_H} \left(\frac{\alpha \beta_H (1-\gamma_H)}{\alpha \beta_H (1-\gamma_H) + (1-\alpha) \beta_L (1-\gamma_L)} \right) \propto (1-\gamma_L)(1-\gamma_H)(1-\alpha) \alpha \beta_L > 0$$

This suggests $\beta_H = 1$. β_L will be defined by

$$\begin{aligned}
0 &= \frac{\alpha (1-\gamma_H)}{\alpha (1-\gamma_H) + (1-\alpha) \beta_L (1-\gamma_L)} - k \\
\beta_L &= \frac{\alpha (1-k)(1-\gamma_H)}{k (1-\alpha)(1-\gamma_L)}
\end{aligned}$$

For future reference the value of the optimization program is $\alpha\left(\frac{k(1-\gamma_L)+(1-k)(1-\gamma_H)}{k(1-\gamma_L)}\right)$.

$\mathcal{P}6(M)$ can be rewritten as:

$$\begin{aligned}
&\max_{\boldsymbol{\beta}} \alpha\beta_H + (1-\alpha)\,\beta_L + q\left(\alpha\left(1-\beta_H\right)\gamma_H + (1-\alpha)\left(1-\beta_L\right)\gamma_L\right) \\
&\text{s.t.} \quad E[\theta|r=g, m=h] \geq k, \\
&\qquad\;\; E[\theta|r=g, m=\emptyset] \geq k, \\
&\qquad\;\; E[\theta|r=b, m=h] \geq k, \\
&\qquad\;\; E[\theta|r=g, m=l] \geq k, \\
&\qquad\;\; 1 \geq \beta_H \geq \beta_L \geq 0.
\end{aligned}$$

The first and second constraints are slack if the third and fourth are satisfied. Hence, the Lagrangean is

$$\begin{aligned}
\mathcal{L}_6 &= \alpha\beta_H + (1-\alpha)\,\beta_L + q\left(\alpha\left(1-\beta_H\right)\gamma_H + (1-\alpha)\left(1-\beta_L\right)\gamma_L\right) \\
&\quad +\mu_1\left(E[\theta|r=b, m=h]-k\right) + \mu_2\left(E[\theta|r=g, m=l]-k\right) \\
&\quad +\mu_3\left(1-\beta_H\right) + \mu_4\left(\beta_H-\beta_L\right) + \mu_5\left(\beta_L\right)
\end{aligned}$$

The Karush-Kuhn-Tucker stationarity conditions are

$$\begin{aligned}
\frac{\partial\mathcal{L}_6}{\partial\beta_H} &= \alpha\left(1-\gamma_H q\right) \\
&\quad +\mu_1\frac{\partial E[\theta|r=b, m=h]}{\partial\beta_H} + \mu_2\frac{\partial E[\theta|r=g, m=l]}{\partial\beta_H} - \mu_3 + \mu_4 = 0 \\
\frac{\partial\mathcal{L}_6}{\partial\beta_L} &= (1-\alpha)\left(1-\gamma_L q\right) \\
&\quad +\mu_1\frac{\partial E[\theta|r=b, m=h]}{\partial\beta_L} + \mu_2\frac{\partial E[\theta|r=g, m=l]}{\partial\beta_L} - \mu_4 + \mu_5 = 0,
\end{aligned}$$

the Karush-Kuhn-Tucker feasibility conditions are

$$\begin{aligned}
\frac{\alpha\left(1-\beta_H\right)\gamma_H}{\alpha\left(1-\beta_H\right)\gamma_H + (1-\alpha)\left(1-\beta_L\right)\gamma_L} - k &\geq 0 \\
\frac{\alpha\beta_H\left(1-\gamma_H\right)}{\alpha\beta_H\left(1-\gamma_H\right) + (1-\alpha)\,\beta_L\left(1-\gamma_L\right)} - k &\geq 0 \\
1-\beta_H &\geq 0 \\
\beta_H-\beta_L &\geq 0 \\
\beta_L &\geq 0
\end{aligned}$$

and the Karush-Kuhn-Tucker complementarity slackness conditions are

$$\begin{aligned}
\mu_1 \left(E[\theta|r=b,m=h]-k\right) &= 0\\
\mu_2 \left(E[\theta|r=g,m=l]-k\right) &= 0\\
\mu_3 \left(1-\beta_H\right) &= 0\\
\mu_4 \left(\beta_H-\beta_L\right) &= 0\\
\mu_5 \left(\beta_L\right) &= 0
\end{aligned}$$

We know:

1. $\mu_4 = 0$ (and so $\beta_H > \beta_L$) because otherwise, if $\Pr(H|r=g,m=\emptyset)-k \geq 0$ then it has to be that $\Pr(H|r=b,m=\emptyset)-k \geq 0$. But we know that $\Pr(H|r=b,m=\emptyset) < \alpha \Rightarrow \Pr(H|r=b,m=\emptyset)-k < \alpha-k < 0$ which is a contradiction.

2. $\mu_3 = 0$ (and so $\beta_H < 1$), because otherwise $\Pr(H|r=b,m=h) = 0$ which implies $\Pr(H|r=b,m=h)-k<0$ (and contradicts the constraint).

3. Since $\mu_3 = 0$ ($\beta_H < 1$), then it must be true that $\mu_1 > 0$ because otherwise $\frac{\partial \mathcal{L}_6}{\partial \beta_H} > 0$ implying $\beta_H = 1$ which is a contradiction.

4. Since $\mu_4 = 0$, then it must be true that $\mu_2 > 0$ because otherwise $\frac{\partial \mathcal{L}_6}{\partial \beta_L} > 0$ implying $\beta_L = 1$ which contradicts $\beta_L < \beta_H < 1$.

We note that β_H and β_L are defined by the first and second constraints binding:

$$\begin{aligned}
\beta_H &= \frac{\left(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha)\right)(1-\gamma_L)}{\alpha(1-k)(\gamma_H-\gamma_L)}\\
\beta_L &= \frac{\left(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha)\right)(1-\gamma_H)}{k(1-\alpha)(\gamma_H-\gamma_L)}
\end{aligned}$$

We need $\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha) > 0$ for β_H and β_L to be non-negative. If this condition does not hold then $\beta_L = 0$ and

$$\begin{aligned}
\frac{\alpha(1-\beta_H)\gamma_H}{\alpha(1-\beta_H)\gamma_H+(1-\alpha)\gamma_L}-k &= 0\\
\frac{\alpha(1-k)\gamma_H-k(1-\alpha)\gamma_L}{\alpha(1-k)\gamma_H} &= \beta_H
\end{aligned}$$

but because we assumed $\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha)<0$, this implies $\beta_H<0$, which is not feasible. So $\mathcal{P}6(M)$ has a solution only for $\gamma_H>\frac{k(1-\alpha)}{\alpha(1-k)}\gamma_L$. Lastly, we note that $\beta_H<1$ because

$$\begin{aligned}1-\beta_H &= 1-\frac{(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha))(1-\gamma_L)}{\alpha(1-k)(\gamma_H-\gamma_L)}\\ &\propto \alpha(1-k)(\gamma_H-\gamma_L)-(1-\gamma_L)(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha))\\ &= \gamma_L(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha))+\gamma_L(k-\alpha)\\ &> 0,\end{aligned}$$

by assumption. For future reference, the value of the optimization program is:

$$M_D\cdot(k(1-\gamma_L)+(1-k)(1-\gamma_H)+\gamma_H\gamma_L q)+(k-\alpha)\gamma_H\gamma_L q$$

where $M_D\equiv\frac{(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha))}{(\gamma_H-\gamma_L)(1-k)k}$.

Below is a summary of the values of the programs:

(A) If $\alpha<\frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$, then

- (a) The value of $\mathcal{P}1(M)$ is $\frac{q\alpha\gamma_H}{k}$;
- (b) The value of $\mathcal{P}2(M)$ is $\frac{\alpha}{k}(q(k\gamma_H+(1-k)\gamma_L)+(1-q))$;
- (c) $\mathcal{P}3(M)$ is not feasible;
- (d) $\mathcal{P}4(M)$ is not feasible;
- (e) The value of $\mathcal{P}5(M)$ is $\alpha\left(\frac{k(1-\gamma_L)+(1-k)(1-\gamma_H)}{k(1-\gamma_L)}\right)$;
- (f) $\mathcal{P}6(M)$ is not feasible;

(B) If $\alpha>\frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$, then

- (a) $\mathcal{P}1(M)$ is not feasible;
- (b) The value of $\mathcal{P}2(M)$ is $\frac{\alpha}{k}(q(k\gamma_H+(1-k)\gamma_L)+(1-q))$;
- (c) The value of $\mathcal{P}3(M)$ is $q(\alpha\gamma_H+(1-\alpha)\gamma_L)$;
- (d) The value of $\mathcal{P}4(M)$ is $q(\alpha\gamma_H+\gamma_L(1-\alpha))+(1-q)\frac{\alpha(1-k)\gamma_H-k(1-\alpha)\gamma_L}{(1-k)(\gamma_H-\gamma_L)k}$;
- (e) The value of $\mathcal{P}5(M)$ is $\alpha\left(\frac{k(1-\gamma_L)+(1-k)(1-\gamma_H)}{k(1-\gamma_L)}\right)$;

(f) The value of $\mathcal{P}6(M)$ is $M_D{\cdot}(k\,(1-\gamma_L)+(1-k)\,(1-\gamma_H)+\gamma_H\gamma_L q){+}(k-\alpha)\,\gamma_H\gamma_L q$ where $M_D \equiv \frac{(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha))}{(\gamma_H-\gamma_L)(1-k)k}$.

It is immediate that in case (B) the value of program $\mathcal{P}3(M)$ is lower than the value of program $\mathcal{P}4(M)$. So we only need to consider:

(A) If $\alpha < \frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$, the values of $\mathcal{P}1(M)$, $\mathcal{P}2(M)$ and $\mathcal{P}5(M)$.

(B) If $\alpha > \frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$, the values of $\mathcal{P}2(M)$, $\mathcal{P}4(M)$, $\mathcal{P}5(M)$ and $\mathcal{P}6(M)$.

As a last step we consider the case when Condition 1 is satisfied and compare the values of the programs above with the value of program $\mathcal{P}3(D)$ (the highest value program under the discretion regime as shown in Proposition 1). The comparison reveals that the value of program $\mathcal{P}3(D)$ is strictly larger than:

- the values of programs $\mathcal{P}1(M)$ and $\mathcal{P}5(M)$;
- the value of program $\mathcal{P}2(M)$ if $\gamma_H < g_o$, where $g_o \equiv \frac{1-\gamma_L(1-\gamma_L(1-k))q}{1-\gamma_L kq} > 0$;
- the value of program $\mathcal{P}6(M)$ because the value of program $\mathcal{P}6(M)$ is lower than the value of program $\mathcal{P}1(D)$ which is lower than the value of program program $\mathcal{P}3(D)$;
- the value of program $\mathcal{P}4(M)$ because the value of program $\mathcal{P}4(M)$ is lower than the value of program $\mathcal{P}6(D)$ if $\gamma_H < g_{oo}$, where $g_{oo} \equiv \frac{q-\gamma_L(1-(1-q)(1-k))}{1-\gamma_L q-k(1-q)}$. Feasibility requires that $g_{oo} > 0$ which holds when $q > 1-\frac{1-\gamma_L}{1-\gamma_L(1-k)}$. Further, (as we show above) the value of program $\mathcal{P}6(M)$ is lower than the value of program program $\mathcal{P}3(D)$;

It follows that a sufficient condition for discretion to be strictly valuable is that $\gamma_H < \min\{\overline{g}, g_o, g_{oo}\}$ and $q > 1-\frac{1-\gamma_L}{1-\gamma_L(1-k)}$. ■

Proof of Proposition 5:

Item (i): Using the proof of Proposition 4, we consider the case when Conditions 1 and 2 are not satisfied and compare the values of programs $\mathcal{P}1(M)$, $\mathcal{P}2(M)$ and $\mathcal{P}5(M)$ (the relevant programs when Condition 2 is not satisfied) with the value of program $\mathcal{P}4(D)$ (the highest value program under the discretion regime as shown in Proposition 2, item (i)). The comparison reveals that the value of program $\mathcal{P}4(D)$:

- is equal to the value of program $\mathcal{P}1(M)$;
- is strictly larger than the value of program $\mathcal{P}2(M)$ if $\gamma_H > \frac{\gamma_L(1-k)q+(1-q)}{q(1-k)} \equiv g^o$. Feasibility requires that $g^o < 1$ which holds if $q > \frac{1}{2-\gamma_L(1-k)-k}$;
- is strictly larger than the value of program $\mathcal{P}5(M)$

It follows that the firm is indifferent between discretion and mandatory disclosure if $\gamma_H > \max\{\overline{g}, g^o\}$ and $q > \frac{1}{2-\gamma_L(1-k)-k}$.

Item (ii): Using the proof of Proposition 4, we consider the case when Condition 1 is not satisfied but Condition 2 is satisfied and compare the values of programs $\mathcal{P}2(M)$, $\mathcal{P}4(M)$, $\mathcal{P}5(M)$ and $\mathcal{P}6(M)$ (the relevant programs when Condition 2 is satisfied) with the value of program $\mathcal{P}1(D)$ (the highest value program under the discretion regime as shown in Proposition 2, item (ii)). The comparison reveals that the value of program $\mathcal{P}1(D)$ is strictly lower than the value of program $\mathcal{P}4(M)$ if $\gamma_H > \frac{1-\gamma_L}{1-q\gamma_L} \equiv g^{oo}$. We note that $g^{oo} < 1$ (so $\gamma_H > g^{oo}$ is feasible). It immediately follows that the firm strictly prefers mandatory disclosure if $\gamma_H > \max\{\overline{g}, g^{oo}\}$. ■

Proof of Proposition 6: For this proof we follow a more general approach and step away from the binary distribution assumptions of the state, the public reports r and the private signals s. We also allow, implicitly, for any payoff function, requiring only that an optimal reporting system exists. The underlying state is $\theta \in \Theta \in \mathbb{R}$ and the prior beliefs are $\mu_o(\theta) \equiv \{\Pr(\theta)\}_{(\theta\in\Theta)}$.[22] In general, μ denotes beliefs. When the firm has no access to private information (i.e., the pure persuasion benchmark), $\mu_r(\theta) \equiv \Pr(\theta|r)$ is the set of posterior beliefs over θ induced by a public report $r \in R$ where $R \in \mathbb{R}$ is the report space and $\tau_\beta(\mu_r)$ is the distribution of beliefs induced by a reporting system $\beta = \{\Pr(r|\theta)\}_{r\in R}$. For the case when the firm might have access to private information, we suppress the private signal *per se*, and assume a general distribution of messages, which may depend on reports, the suppressed private signals, the underlying state, and the firm's optimizing behavior with respect to messages. That is, we let $\gamma = \{\Pr(m|\theta)\}_{m\in M}$, where $M \in \mathbb{R}$ is the message space. Given messages, $\mu_{r,m}(\theta)$ is the set of posterior beliefs over θ induced by a report-message

[22] In our main specification, $\theta \in \{\theta_H, \theta_L\}$ and $\mu_o(\theta) = \Pr(\theta = \theta_H) = \alpha$. This proof uses discrete distributions. An alternative proof using continuous distributions is available from the authors.

combination $\{r, m\} \in R \times M$ and $\tau_{\beta,\gamma}(\mu_{r,m})$ is the distribution of posterior beliefs induced by the reporting system β and the message based on the private signal system γ. Note that $\mu_{r,m}(\theta)$ is a general form applicable whether the firm has discretion over the disclosure or not. Absent private information, the sender will choose an optimal reporting system $\hat{\beta}$ that induces a distribution over posteriors,

$$\tau_{\hat{\beta}}(\mu) = \sum_{r:\mu_r=\mu} \sum_{\theta\in\Theta} \Pr(r|\theta, \hat{\beta})\mu_o(\theta)$$

and posterior beliefs generated by each report,

$$\mu_r(\theta|\hat{\beta}) = \frac{\Pr(r|\theta, \hat{\beta})\mu_o(\theta)}{\sum_{\theta\in\Theta} \Pr(r|\theta, \hat{\beta})\mu_o(\theta)}.$$

With private information, the sender will choose an optimal reporting system $\bar{\beta}$ that induces

$$\begin{aligned}
\tau_{\bar{\beta},\gamma}(\mu) &= \sum_{r,m:\mu_{r,m}=\mu} \sum_{\theta\in\Theta} \Pr(r, m|\theta, \bar{\beta}, \gamma)\mu_o(\theta) \\
&= \sum_{r,m:\mu_{r,m}=\mu} \sum_{\theta\in\Theta} \Pr(m|\theta, \gamma, r) \Pr(r|\theta, \bar{\beta})\mu_o(\theta)
\end{aligned}$$

and beliefs generated by each report-message combination,

$$\begin{aligned}
\mu_{r,m}(\theta|\bar{\beta}, \gamma) &= \frac{\Pr(r, m|\theta, \bar{\beta}, \gamma)\mu_o(\theta)}{\sum_{\theta\in\Theta} \Pr(r, m|\theta, \bar{\beta}, \gamma)\mu_o(\theta)} \\
&= \frac{\Pr(m|\theta, \gamma, r) \Pr(r|\theta, \bar{\beta})\mu_o(\theta)}{\sum_{\theta\in\Theta} \Pr(m|\theta, \gamma, r) \Pr(r|\theta, \bar{\beta})\mu_o(\theta)}
\end{aligned}$$

Let $dim(i) = \eta_i$, for $i \in \{R, M, \Theta\}$, be the number of possible reports, messages, and states, respectively. Any $\tau_{\bar{\beta},\gamma}(\mu)$ is implementable with only the reporting system, β, as we can set

$$\tau_{\beta'}(\mu) = \tau_{\bar{\beta},\gamma}(\mu), \quad \forall\mu \tag{28}$$

by choosing a reporting system that randomizes over reports. That is, (28) is a system of $\eta_R \cdot \eta_M \cdot \eta_\Theta$ equations, and the firm is only constrained by Bayesian plausibility (see Proposition 1 of Kamenica and Gentzkow, 2011).

With a private signal system γ, the firm can only induce the optimal report-only distribution of beliefs $\tau_{\hat{\beta}}(\mu)$ under special circumstances. Specifically, we must have

$$\begin{aligned}
\tau_{\hat{\beta}}(\mu) &= \sum_{r:\mu_r=\mu} \sum_{\theta\in\Theta} \Pr(r|\theta, \hat{\beta})\mu_o(\theta) \\
&= \sum_{r'',m:\mu_{r'',m}=\mu} \sum_{\theta\in\Theta} \Pr(r'', m|\theta, \beta'', \gamma)\mu_o(\theta) \\
&= \tau_{\beta'',\gamma}(\mu)
\end{aligned}$$

The analysis carries over naturally to situations in which there are multiple optimal report-only distributions of beliefs. Note that $\tau_{\hat{\beta}}(\mu) = \tau_{\beta'',\gamma}(\mu)$ is a set of $\eta_R \cdot \eta_M \cdot \eta_\Theta$ equations. We have the following cases for the number of degrees of freedom available to solve this system of equations:

- If all messages are uninformative, then $\Pr(m|\theta, \gamma, r'') = \Pr(m), \forall m$ and $\tau_{\hat{\beta}}(\mu) = \tau_{\beta'',\gamma}(\mu)$ reduces to a system of $\eta_R \cdot \eta_\Theta$ equations with $\eta_R \cdot \eta_\Theta$ degrees of freedom, since the firm can choose a conditional probability for each $\{r, \theta\}$ combination. Therefore, $\tau_{\hat{\beta}}(\mu)$ is implementable in the presence of an uninformative message. The uninformative message may be achieved either through the underlying private signal being uninformative, or through the firm being able to commit *ex ante* to an uninformative message.

- If the firm can commit *ex ante* to report-dependent messages, then the firm has $\eta_R \cdot \eta_M \cdot \eta_\Theta$ degrees of freedom, since the firm can choose a conditional probability for each $\{r, m, \theta\}$ combination. The firm can therefore induce $\tau_{\hat{\beta}}(\mu)$.

- If the private signal is informative and the firm cannot commit to either report-dependent messages or uninformative messages, then the firm has fewer than $\eta_R \cdot \eta_M \cdot \eta_\Theta$ degrees of freedom, and the $\tau_{\hat{\beta}}(\mu) = \tau_{\beta'',\gamma}(\mu)$ system of equations cannot in general be satisfied.[23] Note that the inability to set message-dependent reports means that the firm cannot employ a randomization strategy to undo the influence of the messages.

- If the firm, based on its payoff function, prefers a perfectly-informative reporting system, then it will choose perfectly-informative reports irrespective of the properties of the private signal and messages. In this case, the private signals and messages are moot.

■

[23]There are possibly knife-edge cases in which the system $\tau_{\hat{\beta}}(\mu) = \tau_{\beta'',\gamma}(\mu)$ has a solution.

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